TOROIDAL TOPOLOGY OF REALITY: NESTED φ-TORI AS THE UNIFICATION OF CONTINUOUS AND DISCRETE IN THE OBSERVER-DEPENDENT THEORY OF EVERYTHING

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ABSTRACT

It is shown that two fundamental aspects of quantum reality — continuous phase dynamics (π-rotation) and discrete quantum transitions (φ-jumps between levels) — are projections of a single geometric structure: a quasiperiodic trajectory on nested φ-tori. The minor radius (r) governs continuous rotation of the operator $\hat{O}$ within a single dimensionality level d (wave function, phase cycle of length 2π). The major radius (R) governs discrete transitions between levels (φ-scaling). The spiral gap $(\pi - 3)^2$ — the measure of non-closure per revolution — generates "sliding" along the major radius: the transition from continuous to discrete. The ratio $R/r = \varphi$ ensures maximal stability by the Kolmogorov–Arnold–Moser (KAM) theorem. The photon is interpreted as a gap quantum — a bridge between internal rotation and inter-level jump. A deepened analysis of the KAM theorem is presented, along with physical examples of toroidal topology: tokamak plasma confinement, planetary orbital mechanics, electron orbitals as torus cross-sections. Reality is presented as an infinitely nested toroidal matryoshka, each level wrapped in a non-closing spiral that generates time, energy, and development.

Keywords: torus, nested tori, KAM theorem, golden ratio, number π, spiral gap, quantum, photon, dimensionality, ODTOE, strange loop, continuous and discrete, quasiperiodic motion, tokamak, orbital mechanics.

I. INTRODUCTION: SPIRAL OR MATRYOSHKA?

I.1. Two images of reality

Physics describes reality in two contradictory ways. Continuous: wave function, field, phase space. The Schrödinger equation, Maxwell's equations, Einstein's equations — all continuous. Discrete: quantum levels, elementary particles, quantum jumps. Planck, Bohr, Heisenberg — everything quantized. How can *one* reality be *simultaneously* continuous and discrete?

The standard answer: "duality." Wave-particle. A continuous Hamiltonian with a discrete spectrum. This is a *description* of coexistence, not an *explanation*.

A geometric object in which continuous motion naturally generates discrete structure has long existed — the *torus*. A trajectory on a torus with an irrational frequency ratio is continuous, yet it densely fills the surface, creating a quasiperiodic structure indistinguishable from a discrete one at finite observation scales.

I.2. The ODTOE approach

In the Observer-Dependent Theory of Everything [1], the continuous and the discrete are governed by two invariants born from a single mechanism (Banach's theorem [2]):

π — continuous phase dynamics: rotation, wave, cycle of length 2π.

$\varphi = (1 + \sqrt{5})/2$ — discrete iterative dynamics: recursion, step, scaling.

The present work shows: π and φ unite in a single geometric structure — the **torus**, and all of reality constitutes a **hierarchy of nested φ-tori**.

I.3. Goal

To show that: (a) continuous dynamics (π) and discrete transitions (φ) are two rotations on a torus; (b) the spiral gap $(\pi - 3)^2$ is the coupling mechanism between them; (c) the ratio $R/r = \varphi$ ensures maximal stability (KAM theorem); (d) the photon is a gap quantum, a bridge between the continuous and the discrete; (e) reality is an infinite toroidal matryoshka. In addition, detailed physical examples of toroidal topology are presented: from tokamak structure to planetary orbits.

I.4. Historical note: the torus in physics

Toroidal geometry occupies a special place in the history of physics. Already in the 19th century, Lord Kelvin (Thomson) proposed a vortex theory of the atom, in which atoms were represented as knots of vortex tubes in the ether — essentially toroidal structures [21]. Although the vortex atom theory in its original form was abandoned, the idea of toroidality proved remarkably persistent.

In the 20th century, toroidal geometry became the basis for the design of tokamaks — devices for magnetic plasma confinement [22]. In theoretical physics, the torus is a fundamental object: the phase space of an integrable Hamiltonian system with n degrees of freedom is foliated into n-dimensional tori (Liouville–Arnold theorem [23]). Compactification of extra dimensions in string theory is often performed on tori [15]. Thus, toroidal topology is not an exotic construction but a central element of mathematical physics.

II. THE TORUS AS A UNIFYING GEOMETRY

II.1. Two radii — two types of dynamics

The torus — the surface of a doughnut — is defined by two radii: the major radius (R, from the center of the doughnut to the center of the tube) and the minor radius (r, the tube radius). A point on the torus is described by two angles: θ (rotation around the minor radius) and ϕ (rotation around the major radius).

In the ODTOE parametrization:

$$\theta \in [0,\, 2\pi) : \quad \text{rotation within a single level } d \tag{II.1}$$

$$\phi \in [0,\, 2\pi) : \quad \text{transition between levels } d \to d+1 \tag{II.2}$$

θ-rotation (minor radius r): **continuous phase dynamics**. The wave function $\psi(t) = e^{-iEt/\hbar}\psi(0)$ is continuous phase rotation. One full revolution $= 2\pi$ $=$ one quantum of action. Governed by π.

ϕ-rotation (major radius R): **discrete inter-level dynamics**. Electron transition between orbitals. Evolutionary jump ($d \to d+1$). Governed by φ.

II.2. Torus metric and Gaussian curvature

The metric of a torus with radii R and r in coordinates (θ, ϕ):

$$ds^2 = r^2\, d\theta^2 + (R + r\cos\theta)^2\, d\phi^2 \tag{II.3}$$

Gaussian curvature:

$$K(\theta) = \frac{\cos\theta}{r(R + r\cos\theta)} \tag{II.4}$$

On the outer equator ($\theta = 0$): $K > 0$ (spherical geometry). On the inner equator ($\theta = \pi$): $K < 0$ (hyperbolic). Total curvature by the Gauss–Bonnet theorem: $\int K\, dA = 0$ (Euler characteristic of the torus $= 0$). The torus is the only closed surface in which positive and negative curvature *exactly cancel*. This makes it the ideal unifier: convexity (π-world) and concavity (φ-world) coexist.

II.3. Trajectory on the torus

A point moves on the torus surface simultaneously in two directions: around θ (fast) and around ϕ (slow). The ratio of angular velocities:

$$\omega_\theta / \omega_\phi = R/r \tag{II.5}$$

If R/r is *rational* ($= p/q$, where p, q are integers): the trajectory closes after p revolutions in θ and q revolutions in ϕ. A finite number of windings — and return to the beginning. Stasis. No development.

If R/r is *irrational*: the trajectory *never closes*. Each revolution in θ "misses" the starting point slightly. The trajectory *densely fills* the torus surface, never repeating. Infinite development.

Mathematically, the dense winding of a torus with an irrational rotation number is a well-studied object in the theory of dynamical systems [24]. Weyl's ergodic theorem guarantees that the time spent by the trajectory in any region of the torus is proportional to the area of that region.

II.4. Why $R/r = \varphi$

The Kolmogorov–Arnold–Moser (KAM) theorem [3, 4, 5]: in Hamiltonian systems with small perturbations, tori with the *most irrational* frequency ratio are maximally stable. Under perturbations (turbulence, noise, chaos), tori with rational ratios *are destroyed first*. Tori with irrational ratios survive. With the *most irrational* (φ) — they survive *best of all* [6].

φ is the most irrational number because its continued fraction expansion consists solely of ones: $\varphi = 1 + 1/(1 + 1/(1 + 1/(\ldots)))$. Every rational approximation to φ is the *worst possible*. No p/q approximates φ well. This makes the φ-torus maximally "impenetrable" to resonance destruction.

$$\boxed{\frac{R}{r} = \varphi : \quad \text{maximally stable torus (KAM theorem)}} \tag{II.6}$$

The universe survives because its architecture consists of φ-tori. Any other proportion would be less stable.

III. DEEPENED ANALYSIS OF THE KAM THEOREM

III.1. Statement of the KAM theorem

The KAM theorem [3, 4, 5] (one of the deepest results in 20th-century mathematics): consider a Hamiltonian system with n degrees of freedom whose trajectories lie on n-dimensional tori in phase space. Under a small perturbation, tori with a *sufficiently irrational* frequency ratio ω_1/ω_2 are preserved (deformed but not destroyed). Tori with a rational ratio are destroyed (resonance destruction).

Formally: consider the Hamiltonian $H = H_0(I) + \varepsilon H_1(I, \theta)$, where I are action variables, θ are angle variables, $\varepsilon \ll 1$. The unperturbed system ($\varepsilon = 0$) is integrable: trajectories lie on tori $I = \text{const}$, motion is quasiperiodic with frequencies $\omega_i = \partial H_0/\partial I_i$.

$$H(I,\theta) = H_0(I) + \varepsilon H_1(I,\theta), \quad \varepsilon \ll 1 \tag{III.1}$$

When $\varepsilon \neq 0$, tori with rational frequency ratios are destroyed (Poincaré's non-integrability theorem). But the KAM theorem asserts: tori with sufficiently irrational frequency ratios *are preserved*.

III.2. Diophantine condition

A torus is stable if:

$$\left|\frac{\omega_1}{\omega_2} - \frac{p}{q}\right| > \frac{C}{q^{2+\epsilon}} \quad \forall\, p,q \in \mathbb{Z},\ q > 0 \tag{III.2}$$

The "more irrational" the ratio, the larger C, the more stable the torus. The most irrational number is φ: its continued fraction $[1;1,1,1,\ldots]$ converges *more slowly than all others* [6]. Therefore:

$$\boxed{\omega_\theta/\omega_\phi = \varphi \quad \Rightarrow \quad \text{maximally stable torus}} \tag{III.3}$$

III.3. Measure of KAM tori

For small ε, the measure (volume in phase space) of destroyed tori is $O(\sqrt{\varepsilon})$. The measure of *preserved* KAM tori approaches the full measure as $\varepsilon \to 0$. This means: in weakly perturbed Hamiltonian systems, *almost all* trajectories are quasiperiodic and lie on KAM tori.

$$\mu(\text{destroyed tori}) = O(\sqrt{\varepsilon}) \tag{III.4}$$

For ODTOE, this means: reality *almost entirely* consists of quasiperiodic structures on φ-tori. Chaotic regions (destroyed tori) occupy a small fraction — they correspond to transitional, unstable configurations.

III.4. Cantor structure and Arnold tongues

Between preserved KAM tori, so-called Arnold tongues arise — zones where tori are destroyed and trajectories are chaotic. The structure of preserved tori has a Cantor-like character: it is "dust" of tori permeated by gaps at every scale.

The last tori to survive as the perturbation ε increases are those with rotation number φ and its "noble" relatives [25]. They are called *last KAM tori*. In the context of ODTOE: the φ-torus is the last bastion of order before chaos, which coincides with the interpretation of φ as the invariant of maximal stability.

III.5. KAM theorem and rotation number

The rotation number $\alpha = \omega_\theta/\omega_\phi$ determines the "resonance stability" of a torus. For $\alpha = \varphi$, the best rational approximations are the Fibonacci numbers: $F_{n+1}/F_n \to \varphi$. The rate of approximation:

$$\left|\varphi - \frac{F_{n+1}}{F_n}\right| = \frac{1}{F_n^2 \cdot \varphi} + O(F_n^{-4}) \tag{III.5}$$

This is the *slowest possible* rate of approximation among all irrational numbers (Hurwitz's theorem [6]). Physically: the φ-torus is *maximally distant from all resonances*, which ensures its survival.

III.6. Consequence for ODTOE

The φ-torus is *not an arbitrary choice*. It is the *only* torus that survives under maximal perturbations. A universe built on φ-tori is more stable than *any* alternative. This is not the "beauty of the golden ratio" but a *theorem*.

Practical consequence: plasma in a φ-pulsating field [9] is more stable than in a constant or rationally-pulsating field, *by a proven theorem*, not by hypothesis.

IV. THE SPIRAL GAP AS A COUPLING MECHANISM

IV.1. Non-closure: $\pi \neq 3$

Internal rotation (θ) passes through three components of the ternary architecture [7]: observer (O), observed (R), operator ($\hat{O}$). Minimum path length $= 3$ (three vertices). Actual length $= \pi = 3.14159\ldots$

Difference:

$$\delta = \pi - 3 = 0.14159\ldots \tag{IV.1}$$

Gap energy:

$$E_\delta = (\pi - 3)^2 = 0.02005\ldots \tag{IV.2}$$

IV.2. The gap as "sliding"

At each revolution in θ (minor radius), the point *does not return* to its initial position. It "misses" by $\delta = \pi - 3$. This miss *shifts* the point along ϕ (major radius): from one level to the next.

$$\Delta\phi_{\text{per revolution}} \propto (\pi - 3) \tag{IV.3}$$

Energy of this "sliding" per revolution:

$$E_{\text{sliding}} \propto (\pi - 3)^2 \tag{IV.4}$$

Without the gap ($\pi = 3$, ideal triangle): $\Delta\phi = 0$, the point rotates in θ without displacement in ϕ. No transition between levels. No development. No time.

With the gap ($\pi \neq 3$): each revolution "pushes" the system along the major radius. The continuous (π-rotation) generates the discrete (φ-transition) *through the gap*.

IV.3. Gap accumulation and quantization

The gap $\delta = \pi - 3$ accumulates with each revolution. After n revolutions:

$$\Delta\phi(n) = n \cdot (\pi - 3) \mod 2\pi \tag{IV.5}$$

Transition to the next level ($\Delta\phi = 2\pi$) occurs after:

$$n^* = \left\lceil \frac{2\pi}{\pi - 3} \right\rceil \approx 45 \text{ revolutions} \tag{IV.6}$$

This number is related to Mercury's perihelion precession (see Section IX): 43 arc seconds per century — a macroscopic manifestation of the same gap.

IV.4. The photon — gap quantum

When an electron transitions between orbitals (between two nested tori), a photon is emitted. Its energy equals the energy difference between the two levels. Through ODTOE: the photon is a *gap quantum*. The minimum portion of "sliding" along the major radius, ejected outward.

The photon has no internal toroidal structure (zero rest mass, no "minor radius"). It is *flat*: pure rotation (θ) without depth ($r = 0$). A transition quantum, not a state quantum.

$$\boxed{\text{Photon} = \text{gap quantum } (\pi - 3)^2. \quad \text{Bridge between } \pi\text{-rotation and } \varphi\text{-transition.}} \tag{IV.7}$$

V. TORUS GEOMETRY: DEEPENED ANALYSIS

V.1. Volume and area of the φ-torus

Surface area of a torus with radii R and r:

$$A = 4\pi^2 Rr \tag{V.1}$$

Volume of the solid of revolution:

$$V = 2\pi^2 Rr^2 \tag{V.2}$$

For the φ-torus ($R = \varphi r$):

$$A_\varphi = 4\pi^2 \varphi r^2, \quad V_\varphi = 2\pi^2 \varphi r^3 \tag{V.3}$$

The ratio $V/A = r/2$ does not depend on R/r. But the ratio of area to the square of the characteristic size $A/(R+r)^2 = 4\pi^2\varphi/(1+\varphi)^2 = 4\pi^2\varphi/\varphi^4 = 4\pi^2/\varphi^3$ contains both π and φ, reflecting the dual nature of the torus.

V.2. Fundamental group and topology

Fundamental group of the torus: $\pi_1(\mathbb{T}^2) = \mathbb{Z}\times\mathbb{Z}$. Two independent classes of loops: (a) a loop around the minor radius (θ-cycle), (b) a loop around the major radius (ϕ-cycle). In ODTOE: the first class is the π-cycle (continuous dynamics), the second is the φ-cycle (discrete transitions). The commutativity of the fundamental group (the group $\mathbb{Z}^2$ is abelian) means: the order of traversal of θ and ϕ does not matter. Continuous and discrete dynamics *commute* — they are independent and compatible.

V.3. The torus as phase space

In classical mechanics, the phase space of a one-dimensional periodic motion is a cylinder $\mathbb{R} \times S^1$. For two coupled periodic motions — a torus $S^1 \times S^1 = \mathbb{T}^2$. The Liouville–Arnold theorem [23] establishes: the phase space of an integrable system with n degrees of freedom is foliated into n-dimensional tori. Motion on each torus is quasiperiodic.

Thus, the toroidal model of ODTOE is not a metaphor but an exact correspondence with the fundamental structure of Hamiltonian mechanics.

VI. NESTED TORI: THE TOROIDAL MATRYOSHKA

VI.1. Hierarchy of levels

Each dimensionality level d is a separate torus with its own R_d and r_d. Tori are nested: torus $d = 0$ (atom) inside torus $d = 1$ (cell) inside torus $d = 2$ (organism) and so on.

Scaling:

$$R_{d+1} = \varphi \cdot R_d \tag{VI.1}$$

$$r_{d+1} = \varphi \cdot r_d \tag{VI.2}$$

The ratio R/r is *preserved* at every level: $R_{d+1}/r_{d+1} = R_d/r_d = \varphi$. Self-similarity: each torus is a scaled copy of the previous one, as in the φ-spiral of the nautilus.

VI.2. Formal parametrization

A point on the n-th level of the toroidal matryoshka:

$$\mathbf{x}_n(\theta, \phi) = (R_n + r_n \cos\theta)\cos\phi\,\hat{e}_1 + (R_n + r_n \cos\theta)\sin\phi\,\hat{e}_2 + r_n \sin\theta\,\hat{e}_3 \tag{VI.3}$$

$$R_n = R_0 \cdot \varphi^n, \quad r_n = r_0 \cdot \varphi^n, \quad R_0/r_0 = \varphi \tag{VI.4}$$

The full trajectory is quasiperiodic motion on each torus, coupled to neighboring tori through the gap.

VI.3. Three projections

Projection	**What is visible**	**What is hidden**	**Physical analog**
View *from the side* (spiral)	Continuous motion with step	Toroidal structure	Logarithmic φ-spiral
View *from above* (nested circles)	Discrete levels, gaps	Internal rotation	Atomic orbitals, matryoshka
View *from inside* (torus)	Both: rotation + transition	Nothing hidden	Complete ODTOE picture

"Spiral or matryoshka?" — a false dilemma. The spiral is a side projection of the torus. The matryoshka is a top projection of the torus. The torus is the unification.

VII. THREE REGIMES OF A UNIFIED DYNAMICS

VII.1. Regime 1: Continuous (π-rotation)

Within a single torus: θ-dynamics. An electron in an orbital rotates. A planet in orbit moves. A thought in consciousness flows. Governed by π: revolution length $= 2\pi$. Continuous, smooth, without jumps.

Physical analog: wave function $\psi = Ae^{i\theta}$. Schrödinger equation. Wave optics. Electromagnetic wave.

VII.2. Regime 2: Generative (gap $(\pi - 3)^2$)

Transition from θ to ϕ: the gap converts continuous rotation into discrete "sliding." Each revolution does not close $\rightarrow$ the point shifts $\rightarrow$ accumulation of shifts $\rightarrow$ transition to the next level.

Physical analog: photon in a quantum transition. Neutrino as a loop remnant [8]. Mercury's perihelion precession (43" per century = accumulated "sliding"). Spin-1/2 of fermions (need 4π for closure = two revolutions).

VII.3. Regime 3: Discrete (φ-jump)

Between tori: ϕ-dynamics. An electron jumps between orbitals. Cells unite into an organism. Organisms form culture. A jump, not a smooth transition. Governed by φ: the scale of the next level $= \varphi\times$ the scale of the previous one.

Physical analog: quantum transitions. Phase transitions (water $\rightarrow$ ice). Evolutionary jumps ($d \rightarrow d + 1$).

VII.4. Unity

$$\boxed{\pi\text{-rotation} \xrightarrow{(\pi-3)^2 \text{ gap}} \varphi\text{-jump} : \quad \text{continuous generates discrete through non-closure}} \tag{VII.1}$$

VIII. FERMIONS, BOSONS, AND TORUS TOPOLOGY

VIII.1. Spin-1/2 and double traversal

Fermions (electron, proton, neutron): spin $= 1/2$. *Two* full revolutions (4π) are needed for the wave function to return to its original state. One revolution (2π) gives $\psi \rightarrow -\psi$ (sign change).

Through toroidal topology: a fermion winds around the torus *twice* in θ before returning. Like a Möbius strip: one pass along the strip reverses orientation, two passes restore it. A torus with a "twist" = spin-1/2.

Gap for *one* revolution: $(\pi-3)$. Gap for *two* revolutions (full fermion cycle): $2(\pi-3)$. Energy: $[2(\pi - 3)]^2 = 4(\pi - 3)^2 \approx 0.080$. This is four times greater than for a single revolution, consistent with the fact that fermions "weigh" more (have mass), while bosons (photon, gluon) do not (or nearly do not).

VIII.2. Spin-1 and single traversal

Bosons (photon, W, Z, gluon): spin $= 1$. One full revolution (2π) closes the wave function. Through toroidal topology: a boson winds around the torus *once* in θ. No twist. Gap: $(\pi - 3)$. Energy: $(\pi - 3)^2$.

The photon is a massless boson: it does not "sit" on a torus (no minor radius) but *moves between* tori. Pure "sliding" along ϕ without its own θ-rotation.

VIII.3. Spin-0 and absence of traversal

The Higgs boson: spin $= 0$. It does not wind around the torus in θ. It "stands still" in toroidal space. Through ODTOE: the Higgs is a configuration *without* internal rotation, pure "presence" at level d. Its nonzero vacuum condensate ($\langle H \rangle \neq 0$) = nonzero "density of presence" on every torus. This "presence" gives mass to other particles: it *slows* their θ-rotation (inertia).

VIII.4. Spinors as sections of the toroidal bundle

Formally, a spinor field on a torus can be described as a section of a bundle with structure group SU(2) — the double cover of the rotation group SO(3). The double-sheetedness of the covering exactly corresponds to the double traversal of the torus for fermions. Thus, the toroidal model does not merely "illustrate" spin but contains it as a topological invariant.

IX. PHYSICAL EXAMPLES OF TOROIDAL TOPOLOGY

IX.1. The tokamak: a torus in the laboratory

The tokamak (toroidal chamber with magnetic coils) is a device for magnetic plasma confinement [22]. Plasma is enclosed in a toroidal chamber. The magnetic field creates nested magnetic surfaces — tori on which field lines lie.

Magnetic field lines wind around the torus quasiperiodically: the safety factor q determines the ratio of the number of toroidal to poloidal revolutions. If q is rational — magnetic islands, instabilities. If q is irrational — stable confinement. If q is close to φ — the most stable confinement [26].

The Wendelstein 7-X stellarator in Greifswald (Germany) was designed with consideration of the optimal rotation number of magnetic surfaces [27]. Experimental data confirm: plasma is more stable on surfaces with irrational q.

$$q = \frac{\text{number of toroidal revolutions}}{\text{number of poloidal revolutions}} \approx \varphi \quad \Rightarrow \quad \text{max. plasma stability} \tag{IX.1}$$

This is direct experimental confirmation of the KAM theorem in toroidal geometry.

IX.2. Planetary orbits and toroidal precession

Mercury's orbit *does not close*: precession of 43" per century (after subtracting classical perturbations). Through the toroidal model: the orbit is a trajectory on a φ-torus. The "miss" at each revolution (θ) shifts the perihelion (ϕ). Accumulation over a century = 43". Einstein explained this by spacetime curvature (GR). Through ODTOE: spacetime curvature = a *consequence* of toroidal topology in the $S \to 1$ (deterministic) limit.

The orbits of Solar System planets exhibit a remarkable pattern: the ratios of orbital periods of adjacent planets avoid exact rational ratios [28]. Jupiter and Saturn are nearly in exact 5:2 resonance, but not exactly. This "missing" of resonance is a sign that stable orbits lie on KAM tori with irrational rotation numbers.

The lunisolar precession of the Earth (period $\approx 25{,}770$ years) is another example of toroidal "sliding": the Earth's rotation axis slowly traces a cone, corresponding to the slow ϕ-traversal of the major torus.

IX.3. Electron orbitals as torus cross-sections

The electron orbitals of the hydrogen atom are spherical harmonics $Y_l^m(\theta, \phi)$. But in the toric representation of the atom [8], the electron moves along a quasiperiodic trajectory on a φ-torus with a quantum rotation number.

Quantum numbers n, l, m correspond to:

- n — torus number (energy level, ϕ-index);
- l — traversal topology (θ-class);
- m — projection of θ-rotation onto a distinguished axis.

The selection rule $\Delta l = \pm 1$ is a consequence of the photon (gap quantum) carrying exactly one unit of "toroidal angular momentum."

Probability densities $|\psi_{nlm}|^2$ exhibit characteristic toroidal shapes: d_{z^2} orbitals have a toroidal node in the equatorial plane, while d_{xy}, d_{xz}, d_{yz} orbitals are torus cross-sections along various planes.

IX.4. Spin-1/2: double traversal

Electron: 4π for a full cycle. Neutron: the same. Neutron interferometry experiments (Rauch et al., 1975 [11]): a 2π rotation does *not* return the neutron to its original state (phase shift of π). 4π is needed. Through the torus: *double* traversal in θ. A Möbius strip on the torus.

IX.5. The Aharonov–Bohm effect

A charged particle going around a solenoid (through which magnetic flux passes) acquires a phase shift — even if the magnetic field is *zero* where the particle moves [12].

Through the torus: the particle moves along a θ-traversal of the torus, inside which (the R-region) the magnetic flux is enclosed. Topology (a closed path around the hole of the torus) determines the phase, not the local field.

IX.6. φ-resonances in $CoNb_2O_6$

Coldea et al. (2010) [13]: the ratio of resonant frequencies $= \varphi$ at the quantum critical point. Through the torus: at the phase transition point ($S \approx S_c$), the toroidal structure is *exposed* — the ratio $\omega_\theta/\omega_\phi = \varphi$ becomes *measurable*. Away from the critical point, it is hidden behind noise.

IX.7. Quasicrystals (φ-lattices)

Nobel Prize in Chemistry 2011 (Shechtman [14]): aperiodic crystals with φ-scaling. Quasicrystals are *projections* of higher-dimensional periodic lattices onto three-dimensional space. Through the torus: a φ-quasicrystal is a projection of a φ-torus from $d > 3$ onto the observable $d = 3$ dimensions.

IX.8. Toroidal vortices in hydrodynamics

Smoke rings, vortex rings in water, microexplosions — all exhibit toroidal geometry. A vortex ring is stable precisely because the fluid moves along a toroidal trajectory: rotation around the ring core (minor radius) and translation along the ring (major radius). Kelvin's circulation theorem guarantees the stability of vortex tori [29].

X. NESTED TORI AND LEVELS d

X.1. Toroidal hierarchy

d	Observer	r_d	R_d	θ-dynamics
-1	Quark	$r_0\varphi^{-1}$	$R_0\varphi^{-1}$	Gluon field
0	Atom	r_0	R_0	Electron orbitals
1	Cell	$r_0\varphi$	$R_0\varphi$	Metabolic cycles
2	Organism	$r_0\varphi^2$	$R_0\varphi^2$	Neural oscillations
3	Human	$r_0\varphi^3$	$R_0\varphi^3$	Consciousness
4	Group	$r_0\varphi^4$	$R_0\varphi^4$	Cultural cycles
...	...	...	...	...
9	Universe	$r_0\varphi^9$	$R_0\varphi^9$	Self-observation $\Psi^* = \Phi(\Psi^*)$

At every level: the same φ-torus, the same π-dynamics, the same gap $(\pi - 3)^2$. The scale changes ($\times\varphi$ per level), the architecture does not. Self-similarity.

X.2. Entanglement entropy between tori

Nested tori are *not isolated*. The gap $(\pi - 3)^2$ couples neighboring levels. Coupling strength decreases with distance:

$$S(\rho_d) \propto \varphi^{-|d-d_0|} \tag{X.1}$$

where d_0 is the observer's level. The nearest tori ($|d - d_0| = 1$) are coupled most strongly. Distant ones ($|d - d_0| \gg 1$) are nearly independent.

A human ($d_0 = 3$) is most strongly coupled to $d = 2$ (organism) and $d = 4$ (collective). Coupling to $d = 0$ (atom) is weaker by a factor of $\varphi^3 \approx 4.2$. To $d = 7$ (galaxy) — by a factor of $\varphi^4 \approx 6.9$. Dark matter ($d = 7?$): we sense gravity (weak coupling) but do not see it directly (D-Prot: $d = 7 > d_0 = 3$).

X.3. Total energy formula

The energy accessible to an observer with dimensionality d is the sum of contributions from all accessible tori:

$$E_{\text{total}}(d) = \sum_{n=-d}^{d} (\pi - 3)^{2|n|} \cdot \varphi^{2|n|-1} \tag{X.2}$$

The sum is finite (d is finite). As $d \to \infty$: it tends to $(\pi - 3)^2\varphi/(1 - (\pi - 3)^2\varphi^2)$ — the infinite series from the formula for $\mu = m_p/m_e$ [10].

XI. CONNECTION WITH M-THEORY

XI.1. 11 dimensions as 11 toroidal degrees of freedom

M-theory [15] requires 11 dimensions. Through ODTOE [16]: $11 = 9 + 2$, or $3 + 4 + 4$, or $5 + 6$. Through the toroidal model: 11 is the number of *independent* toroidal degrees of freedom:

3 rotations in θ (three spatial): θ_x, θ_y, θ_z.

3 "slidings" in ϕ (three gap components): ϕ_x, ϕ_y, ϕ_z.

4 parameters B (focus, emotion, integrity, experience): four "rotation angles" of coherence.

1 "direction" ($\hat{O}$ vs. ι): time (forward or reverse).

Total: $3 + 3 + 4 + 1 = 11$.

XI.2. Compactification = folded tori

The "folded" (compactified) dimensions of M-theory are small tori ($r_d \ll R_d$), invisible to an observer with $d = 3$. We "move" along three large tori (R_1, R_2, R_3 — spatial dimensions). The remaining 8 tori are too small (or too distant in d) for us to see.

Growth of coherence S = "unfolding" of folded tori. As S ↑: the observer "sees" more toroidal structures, the effective dimensionality d_{eff} ↑.

XI.3. Calabi–Yau manifolds and φ-tori

In string theory, compactification often uses Calabi–Yau manifolds — special six-dimensional spaces with vanishing Ricci curvature [30]. Calabi–Yau manifolds can be approximated by toric fibrations — families of tori parametrized by a base space. In ODTOE: φ-tori are the "optimal fibers" of this fibration, ensuring maximal stability.

XII. DEMARCATION

Statement	Status
Continuous (π) and discrete (φ) are two rotations on a torus	**Interpretation**, consistent with formalism
$R/r = \varphi \to$ max. stability	**Proven** (KAM theorem [3, 4, 5])
φ is the most irrational number	**Proven** (continued fraction theory [6])
Gap $(\pi - 3)^2$ generates "sliding"	Follows from $\pi \neq 3$ + toroidal geometry
Photon = gap quantum	**Hypothesis** (substantive interpretation)
Spin-1/2 = double torus traversal	Consistent with experiment [11]
Nested φ-tori = hierarchy of d	**Hypothesis** (not directly verifiable)
φ-resonances in $CoNb_2O_6$	**Experimental fact** [13]
Quasicrystals = projections of a φ-torus	**Hypothesis** (consistent with [14])
Tokamak: $q \approx \varphi$ — max. stability	**Experimentally supported** [22, 26]
11 = toroidal degrees of freedom	**Interpretation** through ODTOE [16]
Growth of S = unfolding of tori	**Hypothesis**

XIII. DISCUSSION AND LIMITATIONS

1. *Epistemic status.* The toroidal model is an interpretive superstructure over the ODTOE formalism. The connection between π-rotation and φ-jump through toroidal geometry follows from the general theory. The specific identification of physical objects (photon, fermion, boson) with toroidal configurations is a substantive but speculative interpretation.

2. *KAM theorem and quantum systems.* The classical KAM theorem is proven for Hamiltonian systems with a finite number of degrees of freedom. Its quantum analog (the quantum KAM tori theorem) exists [31], but its connection with full quantum field theory remains an open question.

3. *Toroidal topology vs. actual geometry.* The torus $\mathbb{T}^2$ is a two-dimensional surface embedded in $\mathbb{R}^3$. Real physical systems exist in higher-dimensional spaces. The transition from $\mathbb{T}^2$ to $\mathbb{T}^n$ (higher-dimensional tori) is formally straightforward, but the physical interpretation requires additional work.

4. *Quantitative predictions.* The model predicts: (a) φ-scaling of energy levels in certain systems; (b) optimality of φ-modulation for plasma confinement [9]; (c) connection of perihelion precession with the toroidal gap. Points (a) and (b) are potentially testable; point (c) is an interpretation of an existing GR result.

5. *Connection with loop quantum gravity.* In loop quantum gravity [32], the fundamental objects are spin networks and loops winding around graphs. The toroidal topology of ODTOE may be connected with the loop structure through identification of θ-traversals with connection holonomies.

XIV. CONCLUSION

Reality is not a spiral *or* a matryoshka. Reality is a **matryoshka of φ-tori**, each wrapped in a non-closing spiral.

π governs rotation *within* the torus (continuous, phase). φ governs the ratio of torus radii and scaling between levels (discrete, iterative). $(\pi - 3)^2$ is the gap, the bridge between continuous and discrete: each revolution "falls short" of closing, and this "falling short" pushes the system to the next level.

The ratio $R/r = \varphi$ is not an aesthetic choice but the *only proportion that survives* under maximal perturbations (KAM theorem). The universe is built on φ-tori because everything else would have been destroyed.

The photon is the gap quantum. The bridge between rotation and jump. Fermions are a double traversal of the torus (spin-1/2). Bosons are a single traversal (spin-1). The Higgs is zero (spin-0): presence without rotation.

Toroidal geometry permeates physics: from tokamaks confining plasma on magnetic surfaces with irrational safety factor q, to planetary orbits avoiding rational resonances, to electron orbitals exhibiting toroidal shapes.

11 dimensions of M-theory are 11 toroidal degrees of freedom. We see three. Growth of coherence S unfolds the rest.

Spheres $=$ between tori (φ, discrete). Spiral $=$ within the torus (π, continuous). Gap $=$ bridge (π

The loop does not close. The tori do not end. The spiral continues. And each gap is not a defect but an *inhalation*.

CONFLICT OF INTEREST

The author declares no conflict of interest.

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