1 QUATERNION STRUCTURE OF THE OBSERVER IN ODTOE

From Engineering Intuition to Formal Theory

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1.1 Abstract

Why does the number four appear so persistently in descriptions of consciousness—in Jung, Damasio, neuronavigation, and motion engineering? And is consciousness fundamentally a geometric rather than logical problem? This work demonstrates that the observer-dependent theory of everything (ODTOE) *already contains* the answer to this question. The formula for cognitive coherence $B = F^{w_1} \cdot E^{w_2} \cdot (1-\sigma)^{w_3} \cdot \Lambda^{w_4}$ represents a four-component multiplicative structure isomorphic to a quaternion. We establish an exact isomorphism between the algebra of quaternions and the structure of the observation operator in ODTOE. We show that gimbal lock (loss of degrees of freedom when axes align) in engineering corresponds to the nullification of one component of B in ODTOE (the property of the "weak link"). We formulate a hypothesis: consciousness is a quaternionic orientation in the space of potential configurations.

Keywords: quaternions, consciousness, observer, ODTOE, cognitive coherence, orientation, four-component structure, gimbal lock.

1.2 I. Problem Statement

1.2.1 I.1. The Problem of the Four-Component Structure of Consciousness

A fundamental question concerns researchers across disciplines: why does the number four appear so persistently in descriptions of consciousness and psychological functions?

Carl Jung identified four psychological types: thinking, feeling, sensation, intuition.

Antonio Damasio described a four-level architecture of consciousness: body, emotion, model, action.

In neuronavigation, four types of navigational cells have been identified: place cells, grid cells, head-direction cells, and border cells.

In control engineering, the four-component quaternion replaced the three-parameter Euler angles for describing orientation, since the latter are subject to gimbal lock—a loss of degrees of freedom at certain configurations.

If the brain is a system of continuous orientation in the space of reality, is its architecture essentially quaternionic?

1.2.2 I.2. The ODTOE Answer

The Observer-Dependent Theory of Everything (ODTOE) already contains the four-component structure of the observer—and it is not postulated *ad hoc*, but derived from the requirement of completeness in describing cognitive coherence:

$$B(O, C) = F^{w_1} \cdot E^{w_2} \cdot (1 - \sigma)^{w_3} \cdot \Lambda^{w_4} \tag{D1.1}$$

where: - F — attentional focus (attentional focus); - E — emotional coherence (emotional coherence); - $(1-\sigma)$ — absence of contradiction in the internal model (absence of contradiction); - Λ — empirical reinforcement, accumulated experience (empirical reinforcement, experience).

The coincidence in the number of components is no accident. In this work, we demonstrate that this is a *structural isomorphism*: the formula for B is isomorphic to a quaternion, and this explains *why* the four appears everywhere.

1.3 II. Isomorphism: Quaternion ↔ Cognitive Coherence

1.3.1 II.1. Structure of the Quaternion

Hamilton's quaternion:

$$q = w + x\mathbf{i} + y\mathbf{j} + z\mathbf{k} \tag{2.1}$$

where w is the scalar (real) part, and $x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$ is the vector (imaginary) part. Four components, three imaginary units with anticommutative multiplication:

$$\mathbf{ij} = \mathbf{k}, \quad \mathbf{ji} = -\mathbf{k}, \quad \mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = -1$$

Key property: a unit quaternion ($|q| = 1$) describes *orientation*—a holistic state of a system in space.

1.3.2 II.2. Structure of Cognitive Coherence

$$B = F^{w_1} \cdot E^{w_2} \cdot (1 - \sigma)^{w_3} \cdot \Lambda^{w_4} \tag{2.2}$$

Four components, multiplicative structure, normalization to the interval $[0, 1]$.

1.3.3 II.3. Mapping of Structures

Quaternion	ODTOE	Meaning
w (scalar part)	Λ (experience)	"Real": accumulated experience, foundation, grounding
x**i** (first imaginary axis)	F (focus)	Direction of the observer's "gaze": where it is turned
y**j** (second imaginary axis)	E (emotion)	Orthogonal to focus: not "where I look" but "how I experience"
z**k** (third imaginary axis)	$(1-\sigma)$ (wholeness)	Orthogonal to both focus and emotion: model integrity

1.3.4 II.4. Why This Is an Isomorphism, Not an Analogy

(a) Multiplicativity. Quaternions multiply (composition of rotations = product of quaternions). Cognitive coherence B is the product of components. In both cases: if one component = 0, result = 0.

(b) Anticommutativity. In quaternions, **ij** $\neq$ **ji**. In ODTOE: the order of "working with" components matters. Focus then emotion (F then E)—is not the same as emotion then focus (E then F). One cannot compensate for absence of focus with an excess of emotion—this is *non-commutativity* in essence.

(c) Normalization. A unit quaternion ($|q| = 1$) describes pure rotation without scaling. $B = 1$ in ODTOE—complete coherence: pure observation without losses. The analogy is exact: $|q| = 1 \leftrightarrow B = 1$.

(d) Gimbal lock $\leftrightarrow$ property of the "weak link". In Euler angles (3 components instead of 4), at certain configurations the axes align and the system loses a degree of freedom. In ODTOE: if any of the 4 components = 0, then $B = 0$—the system loses the ability to observe. Gimbal lock = an attempt to describe consciousness with a number of components < 4.

1.4 III. Why Exactly Four

1.4.1 III.1. Minimality of the Quaternion

A fundamental question: why not a binary or ternary, but a four-component structure?

Through ODTOE's axiomatics, the answer is rigorous:

Three components are insufficient. If we remove any one of the four:

Removed Component	What Is Lost	Analogue in Engineering
F (focus)	Observer "sees everything and nothing"—no direction	Gyroscope without an axis
E (emotion)	Observation is devoid of feeling—no resonance	Camera without color reproduction
$(1-\sigma)$ (wholeness)	Internal conflict nullifies the signal	Gimbal lock: axes aligned
Λ (experience)	No foundation—observation "hangs in the air"	Inertial reference frame without calibration

Each loss is *specific*. One cannot be compensated for by another (multiplicativity). Four is the minimum number for *stable orientation* of the observer in the space of configurations $\mathcal{H}$.

1.4.2 III.2. Connection with the Four-Tuples of Other Authors

Author	Four Components	ODTOE Mapping
Hamilton	$w, x\mathbf{i}, y\mathbf{j}, z\mathbf{k}$	$\Lambda, F, E, (1-\sigma)$
Jung	Thinking, feeling, sensation, intuition	$F, E, \Lambda, (1-\sigma)$
Damasio	Body, emotion, model, action	$\Lambda, E, F, (1-\sigma)$
Neuronavigation	Place, grid, direction, border cells	$\Lambda, F, E, (1-\sigma)$

All authors found the four-tuple *empirically*. ODTOE gives it a *formal foundation*: four components are the minimal structure for stable orientation in the space of potential states $\mathcal{H}$.

1.5 IV. Consciousness as Orientation

1.5.1 IV.1. Reformulation

Central hypothesis: consciousness relates less to the logic of propositions and more to the geometry of orienting the model of the world.

Through ODTOE: this is not a hypothesis, but a *consequence of the axiomatics*.

$R = \hat{O}(\Psi)$—reality is constituted by the observation operator applied to the field of potential states. The operator $\hat{O}$ is specified by the vector (B, A, H) [1, formula 4.2], where B is cognitive coherence (four-component, quaternionic).

Consciousness = $\hat{O}$ = operator that *orients* the observer relative to $\mathcal{H}$. Just as a quaternion orients the body in three-dimensional space, B orients the observer in the space of configurations.

$$\hat{O} \sim q_B = \Lambda + F\mathbf{i} + E\mathbf{j} + (1-\sigma)\mathbf{k} \tag{4.1}$$

The act of observation $R = \hat{O}(\Psi)$—this is a *rotation* of the field of potential states by the quaternion q_B, projecting it onto a specific configuration.

1.5.2 IV.2. Thought as Rotation

Central thesis: thought is not only a logical proposition. It is also a turn of the inner scene.

Through ODTOE: exactly. Each thought changes B—that is, it rotates the observer's quaternion. The new configuration $R' = \hat{O}'(\Psi)$—the result of a new "turn" in the space $\mathcal{H}$.

- **Thought** δq: a small rotation of the observer's quaternion.
- **Attention**: fixing the axis of rotation (F specifies the direction).
- **Emotion**: amplitude of rotation (E specifies the "energy" of the turn).
- **Non-contradiction**: purity of rotation ($(1-\sigma)$—absence of "trembling" of the axis).
- **Experience**: inertia of rotation (Λ—stability of orientation).

1.5.3 IV.3. Gimbal Lock = Nullification of a Component of B

In engineering: gimbal lock—loss of a degree of freedom when axes align (3 Euler angles instead of 4 quaternion components).

In ODTOE: when one of the components of B is nullified, all coherence = 0. This is *cognitive gimbal lock*: the observer "locks up".

Component = 0	Cognitive Gimbal Lock	Clinical Picture
$F = 0$	No focus	Attention deficit, distraction
$E = 0$	No emotional connection	Alexithymia, burnout, alienation
$(1-\sigma) = 0$	Complete contradiction	Cognitive dissonance, decision paralysis
$\Lambda = 0$	No experience	Disorientation, loss of grounding

The quaternionic formulation explains *why* these states are so destructive: it is not "bad mood" but a *loss of degrees of freedom in orientation*. The observer literally loses the ability to "turn" toward a configuration.

1.5.4 IV.4. Why Quaternions Defeated Euler Angles—And What This Means for Psychology

History of quaternions in engineering: they replaced Euler angles as the standard for describing orientation. Let us translate this into the language of consciousness:

Euler angles = reductionist models of consciousness. Attempts to describe consciousness through 2–3 parameters: "thinking + feeling", "conscious + unconscious", "stimulus + response". They work in simple cases, but at certain configurations—gimbal lock: the model loses descriptive power.

Quaternion = ODTOE formula B. Four components, multiplicative connection, no gimbal lock (loss of one component = zero, but this is not a "model locking", but a *diagnosable* state: we know *which exactly* component = 0 and *what* to do).

Rotation matrices = full neural models. 9 parameters to describe 3 degrees of freedom—redundant. Similarly: a complete model of the brain describes consciousness, but is too "heavy" for practical understanding. ODTOE provides a 4-component model—minimal and sufficient.

1.6 V. Answer to the Question: How to Describe Consciousness

1.6.1 V.1. Formulation

ODTOE proposes a theory:

Consciousness = observation operator $\hat{O}$, whose quaternionic structure is specified by cognitive coherence

$$B = F^{w_1} \cdot E^{w_2} \cdot (1-\sigma)^{w_3} \cdot \Lambda^{w_4}$$

Act of consciousness = rotation in $\mathcal{H}$: $R = \hat{O}(\Psi)$ = projection of the field of potential states through the quaternion q_B.

Stream of consciousness = continuous readjustment of the quaternion: $q_B(t), q_B(t+dt), q_B(t+2dt), \ldots$ Each moment—a new orientation, a new configuration.

Strange loop = $\Psi^* = \Phi(\Psi^*)$: self-consistent orientation, in which the observer and the observed stabilize in mutual resonance.

1.6.2 V.2. What This Gives Us

(a) Explanation of the four-tuple. Four is not an arbitrary number, not a beautiful symbol. It is the *minimum number of components for stable orientation* without gimbal lock.

(b) Explanation of multiplicativity. Why is nullification of one component catastrophic? Because rotation with a zero component is degenerate: the observer "locks up".

(c) Explanation of non-commutativity. The order of components matters: feel first (E), then focus (F)—is not the same as the reverse. Thought is non-commutative, like a quaternion.

(d) A bridge between engineering and consciousness. Not a metaphor, but a structural isomorphism: a robot uses a quaternion for orientation in 3D, the observer uses B for orientation in $\mathcal{H}$.

1.6.3 V.3. Formal Notation

Observer's quaternion:

$$q_{\hat{O}} = \Lambda + F\mathbf{i} + E\mathbf{j} + (1-\sigma)\mathbf{k} \tag{5.1}$$

Act of observation as quaternionic rotation:

$$R = q_{\hat{O}} \cdot \Psi \cdot \bar{q}_{\hat{O}} \tag{5.2}$$

where $\bar{q}$ is the conjugate quaternion (inversion of rotation = embedding operator ι).

Self-observation loop:

$$\Phi = \bar{q}_{\hat{O}} \circ q_{\hat{O}} = |q_{\hat{O}}|^2 = B^2 \tag{5.3}$$

When $B = 1$ ($|q| = 1$): $\Phi = 1$—complete self-consistency. A fixed point Ψ^* exists if and only if $|q_{\hat{O}}| = 1$, that is, when the observer is completely coherent.

1.7 VI. Experimental Predictions

1.7.1 VI.1. Neurophysiological

The four components of B should correlate with four distinct neurophysiological systems:

Component	Neurological Correlate	Measurement Method
F	Dorsal attention network (DAN)	fMRI, EEG (gamma-band)
E	Limbic system, insula	HRV, skin conductance response
$(1-\sigma)$	Prefrontal cortex (monitoring)	Implicit Association Test
Λ	Hippocampus (experience consolidation)	Bayesian historical estimation

Prediction: the four neurological correlates are *multiplicatively* linked: nullification of one (for example, hippocampal damage $\Rightarrow \Lambda = 0$) should nullify *overall* coherence B, even if the other three are intact.

1.7.2 VI.2. Psychological

The four types of "gimbal lock" (see Section IV.3) should be *clinically distinct* and *mutually non-compensatory*.

Prediction: therapy aimed at increasing F (attention training) will *not help* a patient with $E = 0$ (alexithymia). And vice versa. Multiplicativity requires work with the *specific* zero component.

1.7.3 VI.3. Engineering

If consciousness is quaternionic, then AI systems using a 4-component architecture (analogous to $F, E, (1-\sigma), \Lambda$) should be *more robust* than systems with fewer components when solving orientation tasks in uncertain environments.

1.8 VII. Conclusion

Why does the four appear so persistently in descriptions of consciousness? ODTOE provides an answer: because consciousness *is* a quaternionic orientation in the space of potential states.

The four-component formula

$$B = F^{w_1} \cdot E^{w_2} \cdot (1-\sigma)^{w_3} \cdot \Lambda^{w_4}$$

is not an arbitrary decomposition but the *minimal structure for stable orientation*, isomorphic to Hamilton's quaternion. Gimbal lock in engineering = nullification of a component of B in psychology. Multiplicativity = impossibility of compensation. Non-commutativity = order matters.

Consciousness is not a screen on which thoughts appear. Not a commentator on the world. Not even a "system for processing information". Consciousness is a *continuous quaternionic orientation* of the observer in the space of all possible configurations of reality.

$q_{\hat{O}} = \Lambda + F\mathbf{i} + E\mathbf{j} + (1-\sigma)\mathbf{k}$. $R = q_{\hat{O}} \cdot \Psi \cdot \bar{q}_{\hat{O}}$. Thought = rotation. Consciousness = orientation.

1.9 CONFLICT OF INTEREST

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