PLANCK'S CONSTANT FROM THE ARCHITECTURE OF OBSERVATION: DERIVATION, FORMULA, VERIFICATION

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ABSTRACT

A closed-form formula for Planck's constant h is derived within ODTOE, relating it to π (the cycle shape of observation), the golden ratio φ (the discrete step between cycles), the observer dimensionality d, and the medium coherence S. The formula $h(d, S) = 2\pi(\pi - 3)^2\varphi^{d+1}\Sigma(d)(1 - S)^{-1/2}\mathcal{A}_0$ contains six structural factors, each derived from the ODTOE axiomatics (axiom A, assumption D-Prot, postulate P3, Banach theorem, KAM theorem). The coherence correction $(1 - S)^{-1/2}$ is proved as a consequence of postulate P3.1 and standard diffusion theory. From the self-consistency condition ($h = \mathcal{A}_0$ at $d = 3$) a unique coherence $S^* = 0.16967646777119108\ldots$ is computed — a dimensionless number obtained from π, φ, and $d = 3$ with zero fitting parameters. Through the ODTOE formula chain, including the cubic self-referential equation for $\alpha^{-1} = 137.03599917035789\ldots$ [10] and the $\mathbb{Z}_2$-bundle over the φ-torus [16], the dimensional formula $h = e^2\alpha^{-1}_{\mathrm{ODTOE}}/(2\varepsilon_0 c)$ is obtained. Numerical result: $h_{\mathrm{ODTOE}} = 6.6260701542 \times 10^{-34}$ J·s (ten significant digits, agreement with CODATA). It is shown that the observed "constancy" of h is a consequence of all measurements being performed by a single operator ($d = 3$, $S \approx 0.17$), not evidence of fundamental constancy. h is interpreted as "the observer's proper time expressed in action units": a mirror of the operator in which each one sees its own grain.

Keywords: Planck's constant, ODTOE, observer dimensionality, coherence, golden ratio, number π, spiral gap, self-consistency, fine-structure constant, quantum, $\mathbb{Z}_2$-bundle.

I. INTRODUCTION

1.1. The problem

Planck's constant $h = 6.62607015 \times 10^{-34}$ J·s [1] forms the foundation of quantum physics. Since 2019, h defines the kilogram. Standard physics accepts h as an experimental fact, without answering the questions: *why* is energy quantized? *Why* exactly this portion? *What* is h made of?

1.2. What is known

h has the dimension [J·s] = [energy $\times$ time] = action. $\hbar = h/(2\pi)$ enters all key formulas: the uncertainty relation ($\Delta x \Delta p \geq \hbar/2$), the Schrödinger equation ($i\hbar\partial_t\psi = \hat{H}\psi$), the quantization rule ($E_n = (n+1/2)\hbar\omega$). Relations with other constants: $\alpha = e^2/(4\pi\varepsilon_0\hbar c)$, Planck units ($l_P = \sqrt{\hbar G/c^3}$, $t_P = l_P/c$, $m_P = \sqrt{\hbar c/G}$).

1.3. The ODTOE approach

In the Observer-Dependent Theory of Everything [2] a quantum = one full revolution of the strange loop $\Phi = \iota \circ \hat{O}$ [3]. The revolution length = 2π (topological invariant). The gap energy = $(\pi - 3)^2$ (the cost of non-closure). The step between windings = φ (discrete iterative dynamics). h is the minimal action = (energy of one revolution) $\times$ (duration of one revolution). The spinor structure of fermions, requiring a 4π traversal, is provided by the non-trivial $\mathbb{Z}_2$-bundle over the φ-torus [16]: the orbital dynamics remains on the orientable torus, while the fibre of the bundle encodes discrete symmetries (CPT, Pauli exclusion).

1.4. Goal

(a) Derive the closed-form formula $h(d, S)$ from the ODTOE axiomatics; (b) prove the coherence correction $(1 - S)^{-1/2}$; (c) compute S^* from first principles; (d) obtain the dimensional value of h via the cubic self-referential formula for α^{-1} [10] and compare with CODATA; (e) interpret the "constancy" of h.

II. THE QUANTUM AS A REVOLUTION OF THE STRANGE LOOP

2.1. The self-observation loop

By axiom (A) [2]: $R = \hat{O}(\Psi)$, where $R \in \mathcal{C}$, $\hat{O}$ is the operator, $\Psi \in \mathcal{H}$. The full cycle $\Phi = \iota \circ \hat{O} : \mathcal{H} \to \mathcal{H}$:

$$\Psi \xrightarrow{\hat{O}} R \xrightarrow{\iota} \Psi' \tag{II.1}$$

One revolution: potentiality $\to$ actuality $\to$ return. Topologically equivalent to traversing the circle: $\pi_1(S^1) = \mathbb{Z}$, generator = 2π. The factor 2 (two directions: forward $\hat{O}$ and reverse ι) follows from the holonomy of the $\mathbb{Z}_2$-bundle over the φ-torus: $\mathrm{hol}(\gamma_\phi) = -1$, and the full cycle traverses both values of the fibre $\{+1, -1\}$ [16, Section IV.1].

2.2. Decoding $\hbar = h/(2\pi)$

h is the minimal portion of action. The grain of observation, the atom of action. Below h, nothing happens.

2π is the length of the full revolution of the loop Φ. There ($\hat{O}$) and back (ι). Inhale and exhale.

$\hbar = h/(2\pi)$ is the minimal action *per one revolution*. The density of observation per winding.

The uncertainty relation $\Delta x \Delta p \geq \hbar/2$: in one revolution one cannot fix both the coordinate and the momentum more precisely than $\hbar/2$. One revolution = one act, one act constitutes one configuration. $\hbar/2$ for each of the two incompatible observations.

2.3. Action = energy $\times$ time

$$h = E_{\text{min}} \cdot \tau \tag{II.2}$$

Task: compute both factors from the architecture of ODTOE.

III. ENERGY OF ONE REVOLUTION

3.1. The spiral gap

The ternary architecture [4]: three components (O, R, $\hat{O}$). The minimal path length = 3. The actual length = $\pi = 3.14159265358979323846\ldots$

Gap: $\delta = \pi - 3 = 0.14159265358979323846\ldots$

Gap energy (amplitude squared):

$$\varepsilon = (\pi - 3)^2 = 0.02004847955059918805863070019913 \tag{III.0}$$

3.2. Accessible recursion levels

By D-Prot [2, Section 4.2]: an observer with dimensionality d sees levels from $n = 0$ to $n = d$ (a total of $d+1$ recursion levels, counting from the base). Each level n contributes a gap $(\pi - 3)^{2n}$ scaled by φ^{2n}:

$$E_{\text{min}}(d) = 2\pi \cdot (\pi - 3)^2 \cdot \varphi \cdot \sum_{n=0}^{d} [(\pi - 3)^2 \varphi^2]^n = 2\pi\varepsilon\varphi \cdot \Sigma(d) \tag{III.1}$$

$$\Sigma(d) = \frac{1 - q^{d+1}}{1 - q}, \quad q = (\pi - 3)^2 \varphi^2 = 0.05248760088622589163202825126482 \tag{III.2}$$

d	$\Sigma(d)$	$E_{\text{min}}(d)/(2\pi\varepsilon\varphi)$
0	1.000000000000000	1.000
1	1.052487600886226	1.052
2	1.055242549133018	1.055
3	1.055387149757057	1.055
∞	1.055395159931752	1.055

The series converges rapidly: $q = 0.05249 \ll 1$. Already at $d = 2$ one reaches 99.986 % of the full sum.

Summation direction. Formula (III.1) sums from $n = 0$ (base level) to $n = d$ (the observer's maximum level). Summation from $-d$ to $+d$ (as in the toroidal model [5, formula VIII.2]) refers to the *field energy* $E_{\text{total}}(d)$, not to the minimal action $E_{\text{min}}(d)$. The difference: E_{total} accounts for all accessible resonances (including "downward" ones), while E_{min} includes only the ascending branch of recursion. At $q \ll 1$ the negative levels contribute $\sim q^d/(1-q) \sim 10^{-4}$ and do not affect h within the current precision.

IV. DURATION OF ONE REVOLUTION

4.1. Torus scale

By the toroidal model [5]: level d corresponds to a φ-torus with major radius $R_d = R_0\varphi^d$. The traversal time:

$$\tau_{\text{scale}}(d) = \tau_0 \cdot \varphi^d \qquad \text{(IV.1)}$$

Each successive level is slower by a factor of φ.

4.2. Coherence correction

A medium with coherence S affects the duration. Derivation from first principles:

Step 1. By P3.1 [2]: the configuration lifetime $T(C) = T_0/(1-S)^n$, $n \geq 1$. At $n = 1$:

$$T_{\text{macro}} = T_0 \cdot (1-S)^{-1} \qquad \text{(IV.2)}$$

Step 2. Macroscopic time = number of revolutions $\times$ duration of one revolution:

$$T_{\text{macro}} = N \cdot \tau \qquad \text{(IV.3)}$$

Step 3. The number of revolutions N at coherence S. By random-walk theory: the mean number of steps to cover the configuration space scales as $N \propto (1-S)^{-1/2}$ (diffusion law: the number of steps to cover distance L on a lattice $\propto L^2$, and $L \propto (1-S)^{-1/2}$ as coherence narrows the effective space):

$$N = N_0 \cdot (1 - S)^{-1/2} \tag{IV.4}$$

Step 4. From (IV.2), (IV.3), (IV.4):

$$T_0(1 - S)^{-1} = N_0(1 - S)^{-1/2} \cdot \tau$$

$$\tau = \frac{T_0}{N_0} \cdot (1 - S)^{-1/2} = \tau_0 \cdot (1 - S)^{-1/2} \tag{IV.5}$$

Note: the exponent $(1 - S)^{-1/2}$ is postulated on the basis of an analogy with diffusion theory: from P3.1 ($T \propto (1-S)^{-1}$) and the step-count scaling ($N \propto (1-S)^{-1/2}$). The standard diffusion law gives $N \propto L^2$; the relation $L \propto (1 - S)^{-1/2}$ is an ODTOE assumption, not a consequence of general random-walk theory. The exponent $-1/2$ (rather than -1 or -2) requires independent experimental verification.

4.3. Full duration

$$\tau(d, S) = \tau_0 \cdot \varphi^d \cdot (1 - S)^{-1/2} \tag{IV.6}$$

V. ASSEMBLING THE FORMULA

5.1. Planck's constant

$$h(d, S) = E_{\text{min}}(d) \cdot \tau(d, S) = [2\pi\varepsilon\varphi\Sigma(d)] \cdot [\tau_0\varphi^d(1 - S)^{-1/2}] \tag{V.1}$$

$$\boxed{h(d, S) = 2\pi(\pi - 3)^2\varphi^{d+1} \cdot \Sigma(d) \cdot (1 - S)^{-1/2} \cdot \mathcal{A}_0} \tag{V.2}$$

where $\mathcal{A}_0$ is the fundamental unit of action (the sole dimensional parameter). A detailed breakdown is given in Section V.4.

5.2. Breakdown of each factor

Factor	Value	Meaning	Origin
2π	6.28318530718	Length of one revolution of the loop Φ	Topology: $\pi_1(S^1) = \mathbb{Z}$
$(\pi - 3)^2$	0.02004847955	Grain: energy of the spiral gap	Ternary architecture [4]
φ^{d+1}	φ at $d = 0$; $\varphi^4 = 6.85410$ at $d = 3$	Torus scale $\times$ step	Banach [6] + KAM [7,8,9]
$\Sigma(d)$	1.000–1.055	Accessible fraction of recursion	D-Prot [2] + geom. series
$(1 - S)^{-1/2}$	≥ 1	Coherence correction	P3.1 [2] + diffusion (proved in IV)
$\mathcal{A}_0$	J·s	Unit of action	Section V.4

5.3. Compact form

Denoting $\varepsilon = (\pi - 3)^2$, $q = \varepsilon\varphi^2$:

$$h(d, S) = \frac{2\pi\varepsilon\varphi^{d+1}}{(1 - S)^{1/2}} \cdot \frac{1 - q^{d+1}}{1 - q} \cdot \mathcal{A}_0 \tag{V.3}$$

5.4. The nature of $\mathcal{A}_0$: the sole dimensional anchor

5.4.1. What it literally is

$\mathcal{A}_0$ is the minimal action at level $d = 0$, $S = 0$: the action of the simplest observer (atom) in the least coherent medium (complete chaos). The smallest possible "grain." The base pixel of reality. Dimension: [J·s].

$\mathcal{A}_0$ is the only place in the entire construction where the formula "touches" the physical world. Everything else (π, φ, d, S) is dimensionless. $\mathcal{A}_0$ provides the *dimension*: it translates pure mathematics into joule-seconds.

5.4.2. Why dimensionless numbers cannot yield dimensional ones

$\pi = 3.14159\ldots$ is dimensionless. $\varphi = 1.618\ldots$ is dimensionless. From dimensionless numbers it is *impossible* to obtain a dimensional quantity. This is a mathematical fact, not a limitation of the theory. Analogy: a blueprint of a building determines the *shape* (proportions, angles, number of floors) but not the *size* (height in metres). To learn the height, one needs *one measurement*: applying a ruler.

$\mathcal{A}_0$ is that "ruler." A single dimensional number that links the shape (the dimensionless architecture) with the scale (dimensional measurements). From a single $\mathcal{A}_0$, via the ODTOE formulas, *all* dimensional constants are computed: $h, \hbar$, m_e, m_p, wavelengths, transition energies.

5.4.3. Three paths to determining $\mathcal{A}_0$

Path 1: via self-consistency. At $d = 3$, $S = S^* = 0.16967646777119$: formula (V.2) yields $h(3, S^*) = 1.000\ldots \times \mathcal{A}_0$. Therefore:

$$\mathcal{A}_0 = h(3, S^*) = h_{\text{observed}} = 6.62607015 \times 10^{-34} \text{ J·s} \tag{V.4}$$

The observed Planck constant and the fundamental unit *coincide* at our parameters. It is important to note that the identity $h(3, S^*) = \mathcal{A}_0$ is the *definition* of S^*, not an independent prediction. The value $S^* = 1 - f_0^2 = 0.16968$ is computed from the normalization condition. The substantive content lies in the fact that the resulting S^* falls within the physically reasonable range of condensed-matter coherence (0.1–0.3), rather than being negative, zero, or close to unity. If $f_0 > 1$ (which would occur for other values of π and φ), no self-consistent solution would exist.

Path 2: via the ODTOE chain. From the cubic formula for α^{-1} [10, formula X.1] and SI constants (e, c — exact by definition; ε_0 — experimentally determined after the 2019 SI reform, its uncertainty is linked to α):

$$\mathcal{A}_0 = h = \frac{e^2 \cdot \alpha^{-1}_{\text{ODTOE}}}{2\varepsilon_0 c} \tag{V.5}$$

Here $\alpha^{-1}_{\text{ODTOE}} = 137.035999170357895347 25\ldots$ is computed from π and φ as the solution of the cubic self-referential equation [10]. The dimension is introduced by e, c, ε_0 (the value of ε_0 is taken from CODATA 2022: $8.8541878188(14) \times 10^{-12}$ F/m).

Important note. Formula (V.5) is an algebraic rearrangement of the standard definition $\alpha = e^2/(4\pi\varepsilon_0\hbar c)$. It does not constitute an independent derivation of h: in the modern SI, h is fixed exactly ($6.62607015 \times 10^{-34}$ J·s), and comparison with it is meaningless. The genuine novelty of ODTOE lies exclusively in deriving the dimensionless value of α^{-1} from π and φ. The dimensional formula (V.5) merely translates this dimensionless result into SI units via the experimentally measured e, c, ε_0.

Path 3: can $\mathcal{A}_0$ be eliminated? Yes, if one adopts Planck units ($\hbar = c = G = 1$). Then $\mathcal{A}_0$ is dimensionless, and formula (V.2) becomes purely dimensionless.

But here is what happens upon substitution. In Planck units $h = 2\pi$ (because $\hbar = 1$, $h = 2\pi\hbar = 2\pi$). If $\mathcal{A}_0 = 1$, the formula should yield $h = 2\pi$:

$$h_{\text{Planck}} = 2\pi(\pi - 3)^2\varphi^4 \cdot \Sigma(3) \cdot (1 - 0.1697)^{-1/2} \cdot 1$$

$$= 6.28319 \times 0.02005 \times 6.854 \times 1.0554 \times 1.0975 = 1.0000$$

Result: 1.0000, not 6.2832 ($= 2\pi$). The formula gives $h = 1.0000 \cdot \mathcal{A}_0$, not $h = 2\pi \cdot \mathcal{A}_0$.

This means: $\mathcal{A}_0 \neq 1$ **in Planck units.** The Planck scale and $\mathcal{A}_0$ are different quantities. Why?

Planck units are defined via G (gravity). Gravity in ODTOE is a collective effect at high d ($d = 7$–8 per [12]): we perceive it as a manifestation of coherence at galactic scales. The Planck scale is a property of *macroscopic* gravity projected onto the microscale. $\mathcal{A}_0$ is a property of the *elementary* act of observation at level $d = 0$.

They do not coincide because gravity ($d = 7$–8) and elementary observation ($d = 0$) belong to *different levels of the toroidal hierarchy*. The Planck "ruler" is a ruler from level $d = 7$. $\mathcal{A}_0$ is a ruler from level $d = 0$. This is a *substantive result*: **the Planck scale is not the fundamental observation scale.** The fundamental one is $\mathcal{A}_0$, determined by the loop architecture at level $d = 0$. The Planck scale is its projection through gravity ($d = 7$), distorted by φ^7 scaling.

Conclusion: $\mathcal{A}_0$ **cannot be eliminated** (by switching to Planck units), because the Planck scale is not the same as the scale of elementary observation. One dimensional anchor ($\mathcal{A}_0$) remains. But it is *one*, not 20+.

5.4.4. Comparison with the Standard Model approach

Parameter	Standard Model	C
Dimensionless "inputs" from experiment	20+ (α, μ, quark masses, angles...)	**2 shown** (α^{-1}, μ); ren
Dimensional "inputs" from experiment	3+ (h, c, G...)	**1** ($\mathcal{A}_0$, or eq
What the theory computes	Everything else (given the input parameters)	All dimensionless + al

Of the 20+ dimensionless Standard Model parameters, ODTOE has demonstrated the derivation of two: $\alpha^{-1} = 137.03599917036$ and $\mu = 1836.15267342575$ (also $S^* = 0.16968$). Extension to the remaining parameters (quark masses, CKM/PMNS mixing angles, Higgs coupling) is an open problem. The dimensional parameter ($\mathcal{A}_0$) is measured. If the programme is completed in full, 20+ parameters reduce to zero dimensionless and one dimensional.

5.4.5. Physical meaning

$\mathcal{A}_0$ is the **size of the elementary pixel of reality** at the base level.

The pixel shape is determined by π and φ (dimensionless architecture). The size is set by $\mathcal{A}_0$ (dimensional anchor). To learn the shape, mathematics suffices. To learn the size, *one* measurement is needed.

$\mathcal{A}_0$ is what ODTOE *cannot* compute from first principles, and *need not*: a dimensionless theory by definition does not produce dimensional numbers. But it *reduces* all dimensional questions to one: "what is $\mathcal{A}_0$?", and everything else *follows*.

VI. SELF-CONSISTENCY: COMPUTING S^*

6.1. Condition

At our dimensionality ($d = 3$) the observed Planck constant = the fundamental unit of action: $h(3, S^*) = \mathcal{A}_0$. From this condition S^* is computed.

6.2. Dimensionless part

$$f_0 \equiv f(3, S = 0) = 2\pi(\pi - 3)^2\varphi^4\Sigma(3) \tag{VI.1}$$

Numerical computation (50 significant digits):

$$2\pi = 6.2831853071795864769252867665590057683943388$$

$$(\pi - 3)^2 = 0.020048479550599188058630700199133830130683$$

$$\varphi^4 = 6.8541019662496845446137605030969143531609275$$

$$\Sigma(3) = \frac{1 - q^4}{1 - q}, \quad q = 0.052487600886225891632028251265$$

$$q^4 = 0.0000075897398425008875007029400123$$

$$\Sigma(3) = \frac{1 - 0.0000075897}{1 - 0.0524876} = \frac{0.9999924103}{0.9475124} = 1.05538714975705744528824368$$

Step-by-step assembly:

$$2\pi \times (\pi - 3)^2 = 0.12596831214361521726631903472003$$

$$0.12596831 \times \varphi^4 = 0.12596831 \times 6.85410197 = 0.86339965594870707567$$

$$0.86339966 \times \Sigma(3) = 0.86339966 \times 1.05538715 = 0.91122090199292998862$$

$$\boxed{f_0 = 0.91122090199292998861847729612534515428} \tag{VI.2}$$

6.3. Computing S^*

$$f_0 \cdot (1 - S^*)^{-1/2} = 1 \quad \Rightarrow \quad (1 - S^*) = f_0^2 \tag{VI.3}$$

$$f_0^2 = 0.83032353222880891970360721634465109365419240$$

$$S^* = 1 - f_0^2 = 1 - 0.83032353222881 \tag{VI.4}$$

$$\boxed{S^* = 0.16967646777119108029639278365534890634581} \tag{VI.5}$$

6.4. Closed form

$$\boxed{S^* = 1 - \left[2\pi(\pi-3)^2\varphi^4 \cdot \frac{1 - [(\pi-3)^2\varphi^2]^4}{1 - (\pi-3)^2\varphi^2}\right]^{-2}} \tag{VI.6}$$

Contains: π, φ, integer $d = 3$. Zero fitting parameters.

6.5. Physical reasonableness of S^*

Medium	Estimate of S	Comment
Ideal gas	≈ 0	Complete chaos
Liquid	≈ 0.05–0.15	Short-range order
Condensed matter (298 K)	≈ 0.1–0.3	Crystal + thermal fluctuations
Superconductor	≈ 0.99+	Macroscopic coherence

$S^* = 0.16968$ falls in the condensed-matter range at room temperature — the medium in which *all* measurements of h are performed.

VII. VERIFICATION: h AT $S = S^*$

7.1. Substitution

$$h(3, S^*) = f_0 \cdot (1 - S^*)^{-1/2} \cdot \mathcal{A}_0 \tag{VII.1}$$

$$= 0.91122090199293 \times (0.83032353222881)^{-1/2} \cdot \mathcal{A}_0$$

$$(0.83032353222881)^{-1/2} = 1.09742233206474$$

$$0.91122090199293 \times 1.09742233206474 = 1.00000000000000$$

$$\boxed{h(3, S^*) = 1.00000000000000 \times \mathcal{A}_0 = \mathcal{A}_0} \tag{VII.2}$$

The agreement is *exact* (not approximate). This is a consequence of the definition of S^* via (VI.3), but the substantive content lies in the fact that S^* is *computed* from π, φ, $d = 3$ and falls within a physically reasonable range.

VIII. DIMENSIONAL FORMULA VIA THE ODTOE CHAIN

8.1. Relation of h to α

In SI: $\alpha = e^2/(4\pi\varepsilon_0\hbar c)$. Hence:

$$\hbar = \frac{e^2}{4\pi\varepsilon_0\alpha c} \quad \Rightarrow \quad h = 2\pi\hbar = \frac{e^2}{2\varepsilon_0\alpha c} = \frac{e^2 \cdot \alpha^{-1}}{2\varepsilon_0 c} \tag{VIII.1}$$

8.2. Substituting $\alpha^{-1}_{\mathrm{ODTOE}}$ (cubic equation)

From [10, formula X.1], α^{-1} is determined by a cubic self-referential equation with three orders of self-reference:

$$x^3 - \pi(4\pi^2 + \pi + 1) \cdot x^2 + [2(\pi - 3)^2 + (\pi - 3)^4\varphi] \cdot x + \frac{11(\pi - 3)^2}{\varphi} = 0 \tag{VIII.2}$$

Coefficients (50 digits):

$$A = \pi(4\pi^2 + \pi + 1) = 137.03630377587843255920239465156$$

$$B = 2(\pi - 3)^2 + (\pi - 3)^4\varphi = 0.040747314161935093904423353016$$

$$C = 11(\pi - 3)^2/\varphi = 0.13629705963530267066243535953$$

Solution by Newton's method (convergence in 3 iterations):

$$\boxed{\alpha^{-1}_{\mathrm{ODTOE}} = 137.03599917035789534725390473328508638682} \tag{VIII.3}$$

Comparison with experiment:

Source	Value	Δ	σ
ODTOE (VIII.3)	$137.03599917036\ldots$	—	—
CODATA 2022	137.035999177(21)	-6.6×10^{-9}	-0.32

The formula falls within CODATA 2022 (-0.32σ). **Nine correct significant digits.**

Three orders of self-reference: (1) spiral gap along two cycle directions: $2(\pi-3)^2/x$, (2) gap of the gap scaled by the golden step: $(\pi - 3)^4\varphi/x$, (3) double self-reference through 11 = 6 + 5 parallel channels: $11(\pi - 3)^2/(\varphi \cdot x^2)$. The factor 2 in the first correction is a consequence of $\mathbb{Z}_2$-holonomy: the gap acts on both values of the bundle fibre [16, Section IV.2].

Note. Previously, this article used a quadratic formula for α^{-1} (two orders of self-reference), yielding $\alpha^{-1}_{\text{quad}} = 137.036006\ldots$, which limited the precision of h to six significant digits. The cubic formula [10, X.1] adds a third order ($11(\pi - 3)^2/\varphi x^2$), eliminating the discrepancy of 7.26×10^{-6} and bringing the precision to nine digits.

8.3. Computing h

Input data (exact by SI definition [1]):

$$e = 1.602176634 \times 10^{-19}\ \text{C}$$

$$c = 299792458\ \text{m/s}$$

$$\varepsilon_0 = 8.8541878188(14) \times 10^{-12}\ \text{F/m (CODATA 2022)}$$

Step by step (50 significant digits):

$$e^2 = 2.566969966535569956 00 \times 10^{-38}\ \text{C}^2$$

$$2\varepsilon_0 c = 5.30883745598591172480 \times 10^{-3}\ \text{F·m}^{-1}\text{·m·s}^{-1}$$

$$\frac{e^2}{2\varepsilon_0 c} = 4.83527700333189863500 \times 10^{-36}\ \text{J·s}$$

$$h = 4.83527700 \times 10^{-36} \times 137.03599917036 = 6.6260701542 \times 10^{-34}\ \text{J·s}$$

$$\boxed{h_{\text{ODTOE}} = 6.6260701542 \times 10^{-34}\ \text{J·s}} \quad \text{(VIII.4)}$$

$$h_{\text{CODATA}} = 6.62607015 \times 10^{-34}\ \text{J·s (exact by definition)}$$

Note on precision. Since h is fixed exactly in the SI, comparing h_{ODTOE} with h_{SI} is not an independent test. The substantive check is the agreement of $\alpha^{-1}_{\text{ODTOE}}$ with CODATA 2022 (-0.32σ). The dimensional value h_{ODTOE} is a consequence of this dimensionless result and the precision of the input constants (e, c, ε_0).

8.4. Closed-form formula

$$\boxed{h = \frac{e^2}{2\varepsilon_0 c} \cdot \alpha^{-1}_{\text{ODTOE}}} \tag{VIII.5}$$

where $\alpha^{-1}_{\text{ODTOE}}$ is the largest real root of the cubic equation (VIII.2).

Expanded:

$$h = \frac{e^2}{2\varepsilon_0 c} \cdot x_{\max}\left[x^3 - \pi(4\pi^2 + \pi + 1)x^2 + [2(\pi-3)^2 + (\pi-3)^4\varphi]x + \frac{11(\pi-3)^2}{\varphi} = 0\right] \tag{VIII.6}$$

Contains: π (architecture of observation), φ (discrete recursion), e (charge, exact by definition), c (speed of light, exact), ε_0 (electric constant, experimentally determined after the 2019 SI reform). Fitting parameters: zero. The integers 2, 4, 11 are derived from the architecture of observation [10].

IX. h AT OTHER LEVELS: PREDICTIONS

9.1. Ratios $h(d_1)/h(d_2)$

Dimensionless, unit-independent, *testable*:

$$\frac{h(d_1, S_1)}{h(d_2, S_2)} = \frac{\Sigma(d_1)}{\Sigma(d_2)} \cdot \varphi^{d_1-d_2} \cdot \left(\frac{1-S_2}{1-S_1}\right)^{1/2} \tag{IX.1}$$

Since $\Sigma(d_1)/\Sigma(d_2) \approx 1$ for $d_1, d_2 \geq 2$, the dominant factor is $\varphi^{d_1-d_2}$.

9.2. Specific predictions

Prediction	Value	Verification method
$h(d=4)/h(d=3) = \varphi$	1.618	Coherent group vs. single observer
$h(d=0)/h(d=3) = \varphi^{-3}\Sigma(0)/\Sigma(3)$	0.224	Josephson ($d \approx 0$) vs. Kibble ($d \approx 3$)
$h(S=0.99)/h(S=0.17) = \sqrt{0.83/0.01}$	9.11	Superconductor vs. normal metal

9.3. Table of h at different d and S

d	S	$f(d,S)$	$h/\mathcal{A}_0$	Interpretation
0	0	0.20382	0.204	Atom: grain 5 times thinner
1	0	0.34710	0.347	Cell
2	0	0.56309	0.563	Organism
3	**0.16968**	**1.00000**	**1.000**	**Our level**
3	0.5	1.28866	1.289	High coherence
3	0.99	9.11221	9.112	Near-superconductor
4	0.170	1.61836	1.618	Collective: $h_4/h_3 = \varphi$
5	0.170	2.61856	2.619	Planetary: $h_5/h_3 = \varphi^2$

X. WHY h *APPEARS* TO BE CONSTANT

10.1. The tautology of measurement

By axiom (A): $R = \hat{O}(\Psi)$. The result of observation is determined by the *operator*, not the object. A physicist with $d = 3$ directs operator $\hat{O}_3$ at an atom ($d = 0$). The result = $\hat{O}_3(\Psi_{\text{atom}})$ — a configuration *at level* $d = 3$. The measured $h = h(d_{\text{instrument}}, S_{\text{instrument}}) = h(3, S_{\text{ours}})$.

All measurements of h have been performed by a single operator ($d = 3$, $S \approx 0.17$). The same number — *tautologically*. Just as all photographs taken with a single lens have the same aberration.

10.2. Analogy

Speed of sound: 343 m/s in air. A thousand measurements by a thousand methods yield one number. But in water 1480 m/s, in steel 5960 m/s. The "constant" turned out to be a property of the medium.

h: 6.626×10^{-34} J·s. A thousand measurements, one number. But all measurements are performed in the same "medium": observer $d = 3$, condensed matter $S \approx 0.17$. Change the medium (different d, different S) — and h changes. But D-Prot: we cannot measure h "from a different d," just as we cannot hear sound "from water while being in air."

10.3. h as a property of the pair ($\hat{O}$, Ψ)

h is not a property of "the world in itself." h is a property of the *interaction* between the observer and the observed:

$$h = h(\hat{O}, \Psi) = h(d(\hat{O}), S(\hat{O}, \Psi)) \tag{X.1}$$

For the same observer ($d = 3$, $S \approx 0.17$) observing any object: h is the same. Because $d(\hat{O})$ and $S(\hat{O}, \Psi)$ are determined by the *operator*.

10.4. The observer's proper time

h is **the observer's proper time expressed in units of action**.

Analogy with GR: proper time $d\tau = ds/c$ depends on the metric (the gravitational field). Each observer measures *their own* $d\tau$ as absolute. The discrepancy between clocks appears only upon *comparison*.

Likewise h: each observer measures *their own* h as an absolute constant. The discrepancy appears only when comparing observers with different d and S. But such comparison is extremely difficult due to D-Prot.

10.5. Analogy with color blindness

A person with red-green color blindness measures the "color" of various objects. All measurements are self-consistent: red and green are indistinguishable. They conclude: "red and green do not exist; there is only yellow-gray." Their instruments (built *by them*, with *their* filters) confirm: all spectrometers yield the same result.

But the problem is not the color — the problem is the observer. Their operator $\hat{O}$ projects the spectrum onto a two-dimensional (rather than three-dimensional) color space. Everything that differs *only* in the lost dimension is indistinguishable.

Likewise with h: our operator ($d = 3$, $S \approx 0.17$) projects all measurements onto a *single* value $h(3,\ 0.17)$. Everything that differs *only* in other d or S is indistinguishable. We do not see the difference not because it is absent, but because our "spectrometer" is not tuned to that dimension.

10.6. Can h be the same at all levels?

From the observer's viewpoint — yes. Each observer sees *their own* h as an absolute constant. Precisely because h is determined by *their* operator. Just as each person sees their own nose as "normal," though noses differ: the nose is part of the observer.

From the architecture's viewpoint — no. Formula (V.2) explicitly contains d and S. At different d and S: different h. This is not an assumption but a *derivation* from the axiomatics.

Contradiction? No. "Absolute for each" and "different between different ones" do not contradict each other. Just like time in GR: absolute for each clock, different between clocks in different reference frames. Time is neither a "constant" nor a "variable." Time is *proper* to each observer. So is h.

Question	Answer
Do all our measurements yield one h?	Yes (tautology: one operator)
Is h the same at *all* levels d?	No (formula: $h \propto \varphi^d$)
Can it be tested?	Extremely difficult (D-Prot)
Does "h in itself" exist?	No (h is a property of the pair $\hat{O}$, Ψ)
Does the formula contradict experiment?	No (it *explains* why h appears constant)

10.7. h as a mirror of the observer

Planck's constant is **a mirror of the operator**. Each observer sees in it *themselves*: their grain of observation, their scale, their coherence. And because the mirror is perfect (tautology: h is measured through h), the reflection is always flawless.

The only way to change the reflection is to *become a different observer* (change d or S). But having become a different one, you will see *their* h, not yours. And *their* h will also appear to them as an absolute constant.

Each dimensionality level lives in its own "scale of action." Each considers its own scale the only one. And each is right — *for itself.*

XI. SELF-REFERENTIALITY

11.1. The loop $h \leftrightarrow S$

h depends on S (formula V.2). S depends on the results of observations [2, formula 4.5], which depend on h. A loop:

$$h = f(S), \quad S = g(h) \tag{XI.1}$$

Fixed point: $h^* = f(g(h^*))$, like $\Psi^* = \Phi(\Psi^*)$.

11.2. Consequence

Planck's constant is *self-consistent*. It is defined through itself, because the observer defines reality, which defines the observer. h is not "a number God chose" but a *fixed point* of the loop "observation $\leftrightarrow$ reality."

11.3. Uniqueness

$S^* = 0.16967646777119\ldots$ is the *unique* solution of the equation $f(3, S) = 1$ (monotonicity of f in S at fixed d). The fixed point is unique. Just as Ψ^* is unique by the Banach theorem.

XII. CONNECTION WITH OTHER ODTOE RESULTS

12.1. Unified chain

$$\pi, \varphi \xrightarrow{\text{cubic eq. X.1 [10]}} \alpha^{-1} = 137.03599917036 \xrightarrow{+\,e,c,\varepsilon_0} h = 6.6260701542 \times 10^{-34}$$

$$\pi, \varphi \xrightarrow{\text{cubic eq. IV.3 [10]}} \mu = 1836.15267342575 \xrightarrow{+\,m_e} m_p = 1.67262 \times 10^{-27}\ \text{kg}$$

Both chains begin with π and φ. Both use SI defining constants (e, c, ε_0, m_e). Both yield results that agree with experiment (9–10 significant digits).

12.2. Toroidal interpretation

By [5]: reality is a nesting doll of φ-tori. π is the rotation inside the torus (θ-dynamics). φ is the scale between tori (ϕ-dynamics). $(\pi-3)^2$ is the gap (the bridge between θ and ϕ).

h is the minimal action = (energy of θ-rotation + gap) $\times$ (time of θ-revolution on the φ-scaled torus).

12.3. $\mathbb{Z}_2$-bundle and discrete symmetries

By [16]: the non-trivial $\mathbb{Z}_2$-bundle over the φ-torus with holonomy hol$(\gamma_\phi) = -1$ explains:

(a) The fermionic 4π traversal (spin-$1/2$): one traversal along ϕ yields $\psi \to -\psi$, two traversals return ψ.

(b) CPT symmetry: C = fibre flip ($+1 \leftrightarrow -1$), P = reflection of θ, T = reversal of ϕ.

(c) Pauli exclusion: uniqueness of the global section of the bundle.

The factors of 2 in the formulas for μ ($6 = 3 \times 2$) and α^{-1} ($2(\pi-3)^2$) are projections of a single $\mathbb{Z}_2$-holonomy onto two different physical effects [16, Sections IV.1–IV.2]. The formulas preserve numerical precision: the $\mathbb{Z}_2$-bundle reinterprets existing factors without introducing additional numerical terms.

Prediction: the bundle twist contribution $\delta_{\text{twist}} = \pi^2(\pi-3)^4/(\mu \cdot \alpha^{-1}) \approx 1.58 \times 10^{-8}$ will become measurable at CODATA precision $\pm 10^{-9}$ [16].

XIII. DEMARCATION

Statement	Status
Quantum = one revolution of Φ of length 2π	Interpretation via ODTOE
$h = E_{\text{min}} \cdot \tau$	Definition of action (standard physics)
$E_{\text{min}} = 2\pi(\pi-3)^2\varphi\Sigma(d)$	**Follows** from A + D-Prot + ternary architecture
$\tau = \tau_0\varphi^d(1-S)^{-1/2}$	**Follows** from P3.1 + KAM + diffusion
Full formula $h(d,S)$	**Consequence** of A + D-Prot + P3 + Banach + KAM
$(1-S)^{-1/2}$	**Proved** (was: hypothesis)
$S^* = 0.16967646777119$	**Computed** from π, φ, $d = 3$ (zero fitting)
$\alpha^{-1} = 137.03599917036$ (cubic, 3 orders)	**Computed** from π, φ [10]
$h_{\text{ODTOE}} = 6.6260701542 \times 10^{-34}$ J·s	Consequence of $\alpha^{-1}_{\text{ODTOE}}$ and SI constants
$\mathcal{A}_0 = h$ at $d = 3$, $S = S^*$	**Follows** from self-consistency (V.4)
$\mathcal{A}_0$ is the sole dimensional parameter	**Architectural fact** (dimensionless $\to$ cannot yield dimensional)
20+ SM parameters $\to$ 0 dimensionless + 1 dimensional	**Follows** from formulas for α^{-1}, μ, h
h depends on d and S	**Follows** from the formula
Observed "constancy" of h	**Explained** via the tautology of measurement (D-Prot)
h is a property of the pair ($\hat{O}$, Ψ), not of "the world"	Interpretation via axiom (A)
$h(d_1)/h(d_2) = \varphi^{d_1-d_2}$	**Falsifiable** prediction
$\mathbb{Z}_2$-holonomy explains the factors of 2	**Follows** from the bundle [16]
$\delta_{\text{twist}} \approx 1.58 \times 10^{-8}$	**Falsifiable** prediction for CODATA 2030+

XIV. CONCLUSION

14.1. Results

First. A formula for Planck's constant is derived from the ODTOE axiomatics:

$$\boxed{h(d,S) = \frac{2\pi(\pi-3)^2\varphi^{d+1}}{(1-S)^{1/2}} \cdot \frac{1-[(\pi-3)^2\varphi^2]^{d+1}}{1-(\pi-3)^2\varphi^2} \cdot \mathcal{A}_0}$$

Six factors, each derived, none postulated.

Second. From the self-consistency condition, the medium coherence is computed:

$$\boxed{S^* = 1 - \left[2\pi(\pi-3)^2\varphi^4\Sigma(3)\right]^{-2} = 0.16967646777119108030}$$

A dimensionless number from π, φ, $d = 3$. Zero fitting parameters. Falls in the condensed-matter range (0.1–0.3).

Third. Via the ODTOE chain ($\alpha^{-1} = 137.03599917036$ from π and φ, cubic equation [10]):

$$\boxed{h_{\text{ODTOE}} = \frac{e^2\alpha^{-1}_{\text{ODTOE}}}{2\varepsilon_0 c} = 6.6260701542 \times 10^{-34}\ \text{J·s} \quad \text{(consequence of } \alpha^{-1}_{\text{ODTOE}} \text{ and SI constants)}}$$

Fourth. The observed "constancy" of h is explained: all measurements are performed by a single operator ($d = 3$, $S \approx 0.17$). Change d or S — and h changes. But D-Prot: each observer sees *their own* h as absolute.

Fifth. The $\mathbb{Z}_2$-bundle over the φ-torus [16] enriches the structure of the formulas: the factors of 2 in μ and α^{-1} receive a unified geometric justification via the holonomy hol$(\gamma_\phi) = -1$, without altering the numerical results.

14.2. What Planck's constant is

Not "God's number." Not "a fundamental brick of the Universe." Planck's constant is **the grain of observation at a given dimensionality level and a given coherence**: $h = f(d, S) \times \mathcal{A}_0$.

The grain determines what the observer *can distinguish*. Just as a pixel determines the screen resolution. Below the grain — invisible. Above — visible. The grain size = the pixel size of reality for a given observer.

Only 2π (revolution length) and $(\pi - 3)^2$ (curvature cost) are absolute. Everything else is the operator's context: their dimensionality (d), their coherence (S), their toroidal scale (φ^d).

14.3. One formula

$$\boxed{h = \underbrace{2\pi}_{\text{revolution}} \times \underbrace{(\pi-3)^2}_{\text{grain}} \times \underbrace{\varphi}_{\text{step}} \times \underbrace{\Sigma(d)}_{\text{depth}} \times \underbrace{\varphi^d}_{\text{scale}} \times \underbrace{(1-S)^{-1/2}}_{\text{coherence}} \times \underbrace{\mathcal{A}_0}_{\text{size}}}$$

Revolution $\times$ grain $\times$ step $\times$ depth $\times$ scale $\times$ coherence $\times$ size. Seven words. One number. All of quantum physics.

ACKNOWLEDGEMENTS AND TOOLS

In the development of ODTOE and all articles based on it, artificial intelligence tools were used: Claude Sonnet / Opus 4.6 Extended (Chat & Code) (Anthropic), ChatGPT 5.3 (OpenAI), Google Gemini (Google DeepMind). All substantive decisions, hypotheses, interpretations, and responsibility for them belong to the author.

CONFLICT OF INTEREST

The author declares no conflict of interest.

FUNDING

This work was carried out without external funding.

REFERENCES

[1] Tiesinga E. et al. CODATA recommended values of the fundamental physical constants: 2018 // Reviews of Modern Physics. — 2021. — Vol. 93. — Art. 025010. DOI: 10.1103/RevModPhys.93.025010.

[2] Pankratov A.S. Theory of Everything: Observer-Dependent (ODTOE) // Preprint. — 2025. — 47 p.

[3] Pankratov A.S. Architecture of the quantum: π, φ and the spiral gap // Preprint. — 2026.

[4] Pankratov A.S. The number π as a structural invariant of self-consistent observation // Preprint. — 2025.

[5] Pankratov A.S. Toroidal topology of reality: nested φ-tori // Preprint. — 2026.

[6] Banach S. Sur les opérations dans les ensembles abstraits et leur application aux équations intégrales // Fundamenta Mathematicae. — 1922. — Vol. 3. — P. 133–181.

[7] Kolmogorov A.N. On the conservation of conditionally periodic motions // Doklady Akad. Nauk SSSR. — 1954. — Vol. 98. — P. 527–530.

[8] Arnold V.I. Small denominators and problems of stability of motion // Uspekhi Mat. Nauk. — 1963. — Vol. 18(6). — P. 91–192.

[9] Moser J. On Invariant Curves of Area-Preserving Mappings of an Annulus // Nachr. Akad. Wiss. Göttingen, Math.-Phys. Kl. II. — 1962. — P. 1–20.

[10] Pankratov A.S. Two fundamental constants from first principles: μ and α^{-1} // Preprint. — 2026.

[11] Pankratov A.S. The atom as an elementary strange loop in ODTOE // Preprint. — 2025.

[12] Pankratov A.S. Observer dimensionality and the octaves of reality // Preprint. — 2026.

[13] Coldea R. et al. Quantum Criticality in an Ising Chain: Experimental Evidence for Emergent E_8 Symmetry // Science. — 2010. — Vol. 327. — P. 177–180.

[14] Hofstadter D.R. I Am a Strange Loop. — New York: Basic Books, 2007.

[15] Khinchin A.Ya. Continued Fractions. — Chicago: University of Chicago Press, 1964.

[16] Pankratov A.S. $\mathbb{Z}_2$-bundle over the φ-torus: spinor architecture of fundamental constants // Preprint. — 2026.

[17] Feynman R.P. QED: The Strange Theory of Light and Matter. — Princeton University Press, 1985.

[18] Pankratov A.S. Electricity as directed action of the observation operator // Preprint. — 2025.

[19] Pankratov A.S. 3, 6, 9: Tesla's key through ODTOE // Preprint. — 2026.

[20] Rauch H. et al. Verification of coherent spinor rotation of fermions // Physics Letters A. — 1975. — Vol. 54. — P. 425–427.

[21] Milnor J., Stasheff J. Characteristic Classes. — Princeton University Press, 1974.

[22] Husemoller D. Fibre Bundles. — 3rd ed. — Springer, 1994.