THE GOLDEN RATIO φ AS AN INVARIANT OF FRACTALITY, SELF-SIMILARITY AND RECURSION IN THE OBSERVER-DEPENDENT THEORY OF EVERYTHING

Anton S. Pankratov
Independent researcher, Kazan, Russia
E-mail: anton.s.pankratov@gmail.com
ORCID: 0009-0002-4870-2995

ABSTRACT

The origin of the golden ratio $\varphi = (1+\sqrt{5})/2$ within the formalism of the Observer-Dependent Theory of Everything (ODTOE) [1] is examined. It is shown that φ is the fixed point of the simplest self-referential map $f(x) = 1 + 1/x$ and constitutes the discrete iterative invariant of the self-observation loop, complementary to the continuous phase invariant π. Three phenomena — recursion, self-similarity, and fractality — are presented as aspects of a single mechanism of iterative self-observation whose formal invariant is φ. The decay of entanglement between levels of ∞-recursion obeys the law $S(\rho_d) \propto \varphi^{-|d-d_0|}$ [3], linking φ to the fractal structure of the observation operator. Experimental signatures are discussed: E_8 symmetry at the quantum critical point of an Ising chain [5], Hardy's probability $P = \varphi^{-5}$ [6], and phyllotaxis in biological systems.

Keywords: golden ratio, self-similarity, fractality, recursion, ODTOE, Banach theorem, KAM theorem, φ-invariant.

I. INTRODUCTION

The golden ratio $\varphi = (1+\sqrt{5})/2 \approx 1.618$ appears in mathematics, physics, and biology so pervasively that its presence is often regarded as an ornamental coincidence. Within the formalism of the Observer-Dependent Theory of Everything (ODTOE) [1], φ receives a structural explanation: it is the discrete iterative invariant of the self-referential dynamics, arising from the same mechanism — the Banach fixed-point theorem [4] — that guarantees the existence of the self-consistent configuration $\Psi^* = \Phi(\Psi^*)$.

If π governs the continuous phase dynamics of the self-observation loop [2], then φ governs its discrete component — the very component that generates fractality, self-similarity, and recursion. These three phenomena are not three separate properties but three facets of a single mechanism: iterative self-observation.

The goal of this paper is to formalize the role of φ in ODTOE, demonstrate its origin from the fixed-point theorem, establish its relationship with π through the complementarity principle of the continuous and the discrete, and discuss experimental signatures.

II. THE GOLDEN RATIO AS A FIXED POINT OF SELF-REFERENCE

II.1. The self-referential equation

The equation $\varphi = 1 + 1/\varphi$ is the simplest nontrivial algebraic equation in which the value is defined through itself. The map $f(x) = 1 + 1/x$ contracts the interval $[3/2, 2]$ with Lipschitz constant $L = 4/9 < 1$, and by the Banach theorem [4] it has a unique positive fixed point:

$$f(x) = 1 + \frac{1}{x} \quad \Longrightarrow \quad x^* = \varphi = \frac{1+\sqrt{5}}{2} \tag{II.1}$$

This result is algebraically analogous to Statement 3 of ODTOE [1]: the theory belongs to the set T of theories whose cardinality it itself determines. The number φ is not a quantity discovered empirically but the inevitable outcome of a self-referential iterative process of minimal complexity.

II.2. Relation to the self-observation map Φ

The ODTOE self-observation map $\Phi(\Psi) = \iota(\hat{O}_\Psi(\Psi))$ contains two components: the forward action $\hat{O} : \mathcal{H} \to \mathcal{C}$ (projection, actualization) and the reverse action $\iota : \mathcal{C} \to \mathcal{H}$ (embedding, return to the field of potential states). The equation $\varphi = 1 + 1/\varphi$ reproduces this architecture in minimal algebraic form: φ (the whole state) $= 1$ (basis) $+1/\varphi$ (the reverse action upon itself). Unity is the minimal act of existence; $1/\varphi$ is the act of self-observation that generates recursion.

III. THREE FACETS OF A SINGLE MECHANISM

III.1. Recursion: φ as the limit of Fibonacci ratios

The Fibonacci sequence $F_n = F_{n-1} + F_{n-2}$ is a discrete analogue of the iterative dynamics of the map Φ. Each step is determined by the two preceding ones, just as the configuration $R_n = \hat{O}(\Psi_n)$ is determined by the field Ψ_n, which is itself the result of the previous observation act $\Psi_n = \iota(R_{n-1})$.

The limit of successive ratios $F_{n+1}/F_n \to \varphi$ expresses the convergence of the iteration orbit to the fixed point. The map $f(x) = 1 + 1/x$ generates the sequence

$x_0 = 1,\ x_1 = 2,\ x_2 = 3/2,\ x_3 = 5/3,\ x_4 = 8/5,\ \ldots \to \varphi$, which exactly reproduces the ratios F_{n+1}/F_n.

The Binet formula $F_n = (\varphi^n - \psi^n)/\sqrt{5}$, where $\psi = (1-\sqrt{5})/2 = -1/\varphi$, explicitly derives the discrete sequence from continuous powers of φ. This is a mirror transition relative to the Wallis formula, in which rational factors generate the transcendental π [2].

III.2. Self-similarity: φ as a scale invariant

Self-similarity in ODTOE is formalized through the principle of recursive self-similarity (∞-embedding): each observable R at level d contains an internal self-consistent configuration Ψ^*_{d-1} that reproduces the ternary architecture at level $d-1$ [1]:

$$\ldots\ \Psi^*_{d-2} \subset \Psi^*_{d-1} \subset \Psi^*_d \subset \Psi^*_{d+1} \subset \Psi^*_{d+2}\ \cdots \tag{III.1}$$

The ternary structure (observer, observable, operator) is reproduced at each level. The transition between levels is an iteration $\Psi^*_d \to \Psi^*_{d-1}$. If the linearization $L = D\Phi|_{\Psi^*}$ has a discrete spectrum with the largest eigenvalue $\lambda_1 = \varphi$ (the eigenvalue of the Fibonacci matrix $M = \left(\begin{smallmatrix}1 & 1\\ 1 & 0\end{smallmatrix}\right)$), then the scaling factor between levels of ∞-recursion is determined by φ.

The decay of entanglement between recursion levels is described by the formula [3]:

$$S(\rho_d) \propto \varphi^{-|d-d_0|} \tag{III.2}$$

where d_0 is the observer's level. Entanglement is maximal at the observer's level and decays exponentially toward remote levels with the characteristic scale $\varphi \approx 1.618$. This is consistent with the D-Prot assumption [1]: the observer does not have access to arbitrarily deep recursion levels.

III.3. Fractality: φ as the invariant of fractal entanglement

The ∞-recursion of ODTOE is a self-similar structure by definition (the ternary architecture is reproduced at each level), and the entanglement of the unified operator $\hat{O}$ between levels inherits fractal properties. The fractal dimension of this structure is determined by φ: it governs the rate at which information decays across scales.

In classical fractal theory, the golden ratio appears as the fractal dimension of a number of self-similar structures — the Fibonacci spiral, the pentagram, and aperiodic Penrose tilings [8]. Within ODTOE this is not coincidental: φ is the unique positive number satisfying $x = 1 + 1/x$, and any structure generated by iterative self-reference inherits it as an invariant.

IV. COMPLEMENTARITY OF π AND φ: CONTINUOUS AND DISCRETE

IV.1. Two aspects of a single dynamics

The two structural invariants of ODTOE do not compete but complement each other [2]:

Aspect	π	φ
Dynamics type	Continuous phase	Discrete iterative
Math. object	Generator $\pi_1(S^1)=\mathbb{Z}$	Fixed point $f(x)=1+1/x$
Number type	Transcendental	Algebraic irrational
Guarantees	Non-closure of phase trajectories	Stability of non-closed orbits
Physical manifestation	$\hbar = h/(2\pi)$, wave functions	Fibonacci numbers, fractals
Role in ODTOE	Length of one full cycle of Φ	Convergence rate to Ψ^*

IV.2. Unity of origin

Both invariants are generated by a single mechanism — the Banach fixed-point theorem [4]. For π: the contracting map on $\mathcal{H}$ guarantees the existence of Ψ^*, and the closure of the loop $\Psi \to \hat{O}(\Psi) \to R \to \iota(R) \to \Psi'$ produces the topological invariant 2π. For φ: the same contracting map, viewed as the discrete iteration $f(x) = 1 + 1/x$, converges to φ.

IV.3. The KAM theorem: why φ is the most stable number

The Kolmogorov–Arnold–Moser theorem [7] establishes that invariant tori with a sufficiently irrational frequency ratio are stable under small perturbations. The golden ratio has the worst rational approximations (continued fraction $\varphi = [1;\, 1, 1, 1, \ldots]$): all partial quotients equal unity, making the convergence of rational approximations to φ the slowest among all irrational numbers.

In the ODTOE context: φ guarantees maximum stability of non-closed orbits near the fixed point Ψ^*. Structures whose scaling is determined by φ are the last to be destroyed when coherence S decreases.

V. EXPERIMENTAL SIGNATURES

V.1. Quantum critical point and E_8 symmetry

At the quantum critical point of the Ising chain $CoNb_2O_6$, the ratio of the two lowest resonance frequencies of magnetic spins equals $\varphi = 1.618\ldots$ — a signature of hidden

E_8 symmetry (Coldea et al., 2010) [5]. From the ODTOE perspective: at the phase transition point (maximum reconfiguration), the discrete iterative invariant of the self-observation system is exposed.

V.2. Hardy's probability

The maximum probability of nonlocal quantum correlation between two particles (Hardy's probability) is $P_{\text{Hardy}} = \varphi^{-5} \approx 0.09017$ [6]. If π normalizes the Gaussian measure in the space of potential states $\mathcal{H}$, then φ sets the fundamental probabilistic limit in quantum nonlocality. The self-consistent observation of two entangled subsystems is bounded by a φ-containing limit.

V.3. Phyllotaxis and biological self-similarity

Leaf divergence angles, petal counts, and spirals of sunflowers and pineapples are all determined by Fibonacci numbers and, consequently, by φ. A living organism in ODTOE is a coherent cluster of observers whose morphogenesis obeys the same iterative mechanism as subatomic recursion. Fibonacci patterns in biology are a macroscopic projection of the φ-invariant.

VI. FORMALIZATION: φ AS THE SKELETON OF ∞-RECURSION

VI.1. Matrix representation

The Fibonacci recurrence relation admits a matrix form:

$$\begin{pmatrix} F_{n+1} \\ F_n \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^n \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{VI.1}$$

The eigenvalues of $M = \left(\begin{smallmatrix} 1 & 1 \\ 1 & 0 \end{smallmatrix}\right)$ are $\lambda_1 = \varphi$ and $\lambda_2 = -1/\varphi$. The largest eigenvalue is φ. If the spectral argument of ODTOE [2] establishes π as the invariant of the continuous spectrum of the linearization of Φ, then φ is the invariant of the discrete spectrum. The two invariants govern two layers of dynamics.

VI.2. Exponential decay of entanglement

The unified operator $\hat{O}$ extends across all levels of ∞-recursion. Its projections $\hat{O}_{d_1}$, $\hat{O}_{d_2}$ onto different levels are not independent. The nonzero von Neumann entropy:

$$S(\rho_d) = -\text{Tr}(\rho_d \log \rho_d), \quad \rho_d = \text{Tr}_{\neq d} |\Psi^*\rangle\langle\Psi^*| \tag{VI.2}$$

indicates entanglement of level d with the rest. The scaling $S(\rho_d) \propto \varphi^{-|d-d_0|}$ means: at the observer's level ($d = d_0$) informational connectivity is maximal; at each step deeper into the recursion, entanglement drops by a factor of $\varphi \approx 1.618$; the exponential decay ensures the D-Prot assumption [1].

VI.3. Relation to configuration inertia

The inertia of a configuration at level d is determined by the sum of observer beliefs: $I(C_d) = \sum w_j \cdot B_j(C_d)$. If entanglement between levels scales as $\varphi^{-|d-d_0|}$, then the effective contribution of deep-level observers to the formation of the configuration at level d_0 decays as $\varphi^{-|d-d_0|}$. Configuration inertia is determined predominantly by the nearest recursion levels, which explains the effectiveness of physical theories operating at their own scale.

VII. DISCUSSION AND LIMITATIONS

1. *Epistemic status.* The origin of φ from the Banach theorem (formula II.1) and the principle of recursive self-similarity (∞-embedding) follow from the ODTOE formalism. The identification of entanglement scaling with $\varphi^{-|d-d_0|}$ (formula III.2) and the interpretation of experimental data (Section V) are speculative and require independent verification.

2. *Quantitative relation between π and φ.* Both invariants are generated by the fixed-point theorem, but a unified formula expressing their quantitative relationship within the full nonlinear dynamics of Φ has not yet been obtained.

3. *Experimental verification.* The law $S(\rho_d) \propto \varphi^{-|d-d_0|}$ predicts a specific rate of correlation decay between hierarchy levels. Direct verification requires entanglement measurements in multi-scale quantum systems.

4. *Relation of φ^{-5} to ODTOE parameters.* Hardy's probability $P = \varphi^{-5}$ [6] matches the ODTOE prediction only at specific values of parameters B, k, S. Establishing this correspondence is an open problem.

VIII. CONCLUSION

The golden ratio is the algebraic skeleton of recursive self-similarity in ODTOE.

Recursion is the iterative dynamics $\Psi_{n+1} = \Phi(\Psi_n)$ converging to Ψ^*; the convergence rate is determined by φ as the largest eigenvalue of the discrete spectrum of the linearization of Φ.

Self-similarity is the reproduction of the ternary architecture at each level d of the ∞-embedding hierarchy; the scaling factor between levels is φ.

Fractality is the self-similar structure of entanglement of the unified operator $\hat{O}$ between recursion levels, with exponential decay $S(\rho_d) \propto \varphi^{-|d-d_0|}$.

All three are manifestations of a single mechanism of iterative self-observation that inevitably generates φ as its discrete invariant, just as the continuous phase dynamics of the same self-observation inevitably generates π [2].

CONFLICT OF INTEREST

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