ETERNAL EXPANSION: TRANSCENDENCE OF π AS PROOF OF THE INEXHAUSTIBILITY OF REALITY

Potentiality pressure on actuality
and the scale factor of the φ-torus
in the Observer-Dependent Theory of Everything

Anton S. Pankratov

Independent researcher, Kazan, Russia

E-mail: anton.s.pankratov@gmail.com
ORCID: 0009-0002-4870-2995

ABSTRACT

The mechanism of the expansion of the Universe is formalized within the toroidal model of ODTOE. It is shown that the φ-torus does not possess a fixed radius in the classical sense: the spiral gap $\delta = \pi - 3$ [15] shifts the trajectory along the ϕ-cycle at every turn of the self-observation loop Φ, increasing the effective scale of the actualized configuration. The Lindemann theorem (1882) on the transcendence of π proves that the gap $\delta = \pi - 3$ is not equal to any rational (or even algebraic) fraction, whence it follows that: (a) the trajectory on the φ-torus does not close for any finite number of turns, (b) the expansion is infinite and inexhaustible. The potentiality pressure of $\mathcal{H}$ (the infinite-dimensional field of unrealized states) on the actualized configuration $\mathcal{C}$ (finite) generates an effective force $\mathcal{F} = (\pi-3)^2 \cdot |\mathcal{H}|/|\mathcal{C}|$ acting at each observation cycle. The structural unattainability of $S = 1$ (full coherence, Ashby's law) guarantees that the pressure never vanishes. A scale factor $a(n) = (1 + \varepsilon/(2\pi\varphi))^n$ is introduced, describing the exponential growth of the effective radius of the φ-torus with the number of observational cycles n. It is shown that the acceleration of expansion ($\ddot{a} > 0$) follows from the positivity $(\pi - 3)^4 > 0$ without invoking the cosmological constant Λ as a free parameter. The agreement of the ODTOE prediction with the Planck 2018 data confirms that the dark energy fraction ($\Omega_\Lambda = \varphi^2/(\varphi^2 + 1 + Z) = 68.86\%$) is a projection of the R-sector of the φ-torus responsible for potentiality pressure.

Keywords: expansion of the Universe, transcendence of π, potentiality pressure, φ-torus, spiral gap, scale factor, dark energy, Ashby's law, ODTOE, KAM theorem.

I. INTRODUCTION

I.1. The problem

The accelerated expansion of the Universe, discovered in 1998 from Type Ia supernovae [1, 2], remains one of the key unsolved problems in physics. The standard model (ΛCDM) describes the expansion through the cosmological constant Λ, whose nature is not derived from first principles. Quantum field theory predicts a vacuum energy $\sim 10^{120}$ orders of magnitude larger than the observed value [3] (the cosmological constant problem). The questions *why* the Universe expands, *why* the expansion is accelerating, and *whether* it will continue have no answer within ΛCDM.

I.2. The ODTOE approach

In the Observer-Dependent Theory of Everything [4], reality R is constituted by the observation operator $\hat{O}$ from the infinite-dimensional field of potential states $\mathcal{H}$: $R = \hat{O}(\Psi)$, $\Psi \in \mathcal{H}$. The actualized configuration $\mathcal{C}$ is always finite, whereas $|\mathcal{H}| = \infty$. The toroidal model [5] represents reality as a hierarchy of nested φ-tori. It has been shown previously [6, 16] that the three topological sectors of the φ-torus generate the cosmological fractions $\Omega_\Lambda : \Omega_{\rm DM} : \Omega_b$, which agree with the Planck 2018 data [7] within 1σ.

The present work formalizes the expansion mechanism: *why* the φ-torus expands, *why* the expansion is accelerated, and *why* it is eternal. The answer to all three questions is a single theorem: π is transcendental.

I.3. Objective

(a) Prove that the expansion of the Universe is eternal and inexhaustible as a mathematical consequence of the transcendence of π (Lindemann's theorem). (b) Formalize the potentiality pressure of $\mathcal{H}$ on the actualized configuration $\mathcal{C}$ through the spiral gap. (c) Show that the φ-torus does not possess a fixed radius: the effective scale grows at each observation cycle. (d) Derive the scale factor and show that the acceleration ($\ddot{a} > 0$) follows from $(\pi - 3)^4 > 0$.

II. TRANSCENDENCE OF π AND NON-CLOSURE OF THE LOOP

II.1. Lindemann's theorem

In 1882 Ferdinand von Lindemann proved [8]: the number π is *transcendental*, that is, it is not a root of any nonzero polynomial with integer coefficients [17, 18]. From the transcendence of π it follows that squaring the circle is impossible: one cannot construct a square equal in area to a given circle using only compass and straightedge.

For ODTOE, three corollaries are essential.

Corollary 1. $\delta = \pi - 3$ is transcendental. *Proof*: if δ were algebraic, then $\pi = \delta + 3$ — the sum of an algebraic and a rational number — would be algebraic. Contradiction with Lindemann's theorem. $\square$

Corollary 2. $N \cdot \delta$ is irrational for every integer $N \neq 0$. *Proof:* if $N\delta = p/q$ for some integers p, q, then $\delta = p/(Nq)$ would be rational, and rational numbers are algebraic. Contradiction with Corollary 1. □

Corollary 3. $N \cdot \delta \neq 2\pi k$ for any integers N, k with $N \neq 0$. *Proof:* if $N(\pi - 3) = 2\pi k$, then $N\pi - 3N = 2\pi k$, whence $\pi(N - 2k) = 3N$, i.e. $\pi = 3N/(N - 2k)$ — a rational number. Contradiction. □

II.2. Physical significance

Corollary 3 is the precise statement of *non-closure* of the trajectory on the φ-torus. At each revolution along θ (minor radius, continuous π-dynamics), the point shifts by $\delta = \pi - 3$ along ϕ (major radius, discrete φ-dynamics) [5, formula III.3]. After N revolutions the cumulative shift is $N\delta$.

If π were rational (or an algebraic irrational of the form p/q): after q revolutions $q\delta = q(\pi - 3)$ could become a multiple of 2π, and the loop would close. Evolution would halt.

The transcendence of π *forbids* this. For no finite number of revolutions does the cumulative shift become a multiple of 2π. The loop *never* closes. The expansion *never* ceases.

II.2a. Connection with Weyl's equidistribution theorem

Weyl's theorem (1916) [25] states: if α is irrational, then the sequence $\{n\alpha\}$ ($n = 1, 2, 3, \ldots$) is uniformly distributed modulo 1 on the interval $[0, 1)$. For $\alpha = \delta/(2\pi) = (\pi - 3)/(2\pi)$ — an irrational (and even transcendental) number — this means that the angular positions of the point on the ϕ-cycle after n revolutions along θ *uniformly fill* the entire ϕ-cycle.

Physical significance: not only does the trajectory not close (Corollary 3), but the coverage of the toroidal surface is *dense*. As $n \to \infty$, the trajectory covers the torus surface *everywhere densely*, meaning that every point on the torus surface lies arbitrarily close to the observation trajectory. This ensures the *completeness* of actualization — every region of potential space is eventually "visited" by the observation operator.

The transcendence of δ strengthens Weyl's result: the rate of equidistribution for transcendental numbers is, as a rule, higher than for algebraic irrationals. The number $\delta = \pi - 3$ possesses good equidistribution properties in the sense of Weyl, which means efficient exploration of potential space by the observational trajectory.

II.3. Theorem on the eternity of expansion

Theorem 1. *If the ratio of the revolution length to the minimal closed path is transcendental ($\pi/3$ is transcendental), then the trajectory on the φ-torus does not close for any finite number of revolutions.*

Proof: closure after N revolutions along θ and M revolutions along ϕ requires:

$$N \cdot \pi = 3N + 2\pi M \tag{II.1}$$

whence $\pi(N - 2M) = 3N$, i.e. $\pi = 3N/(N - 2M)$ — rational. Contradiction with Lindemann's theorem. □

Corollary. The expansion generated by the spiral gap is *eternal* and *inexhaustible*: for the expansion to cease, π would have to become rational, which is mathematically impossible.

Within the φ-torus model, this is not a *hypothesis* about the eternity of expansion — it is a *theorem*. The mathematical part (non-closure of the trajectory) is proved with the same rigor as the impossibility of squaring the circle — both are consequences of the transcendence of π. The physical interpretation (non-closure = expansion of the Universe) depends on acceptance of the toroidal ODTOE model.

II.4. Remark on the roles of π and φ

In the ODTOE formalism, the numbers π and φ play complementary roles [26]: π is the invariant of the continuous phase dynamics (θ-cycle), φ is the invariant of the discrete iterative dynamics (ϕ-cycle). The spiral gap $\delta = \pi - 3$ arises *at the intersection* of the two dynamics: the continuous revolution (2π in θ) does not fit into an integer number of discrete steps (multiples of $2\pi/3$ in ϕ), and the "remainder" $\pi - 3$ is carried over to the next cycle.

The number 3 here is not arbitrary: it corresponds to the minimal number of vertices of a closed polygon (a triangle), i.e. the minimal discrete approximation of a circle. The gap $\delta = \pi - 3$ is a measure of the *incommensurability* of the continuous (π) with the discrete (3), and it is precisely this incommensurability that drives expansion.

We emphasize: the transcendence of π is not an interpretation but a rigorously proved mathematical fact (Lindemann's theorem, 1882 [8]). The entire chain of deductions (Corollaries 1–3, Theorem 1) is built on this fact and on the toroidal model [5]. It is the model, not the mathematics, that is subject to experimental verification.

III. POTENTIALITY PRESSURE ON ACTUALITY

III.1. Field and configuration

By axiom (A) [4]: $R = \hat{O}(\Psi)$. The field of potential states $\mathcal{H}$ is infinite-dimensional. The actualized configuration $\mathcal{C}$ is finite: it is described by a finite set of parameters (d, S, coordinates, momenta). Between $|\mathcal{H}|$ and $|\mathcal{C}|$ there exists an infinite difference:

$$|\mathcal{H}| = \aleph_{\geq 1}, \quad |\mathcal{C}| < \aleph_0 \tag{III.1}$$

All states in $\mathcal{H}$ that are *not* actualized in $\mathcal{C}$ constitute the *unrealized potential*. Its cardinality $|\mathcal{H} \setminus \mathcal{C}| = |\mathcal{H}|$ (subtracting a finite set from an infinite one does not reduce the cardinality).

III.2. Pressure mechanism

Each observation cycle $\Phi = \iota \circ \hat{O}$ actualizes *one* configuration C_n from the *infinite* field $\mathcal{H}$. Unrealized states do not vanish — they remain in $\mathcal{H}$ and "compete" for actualization on the next cycle. This competition creates *pressure* — the tendency of the field to realize itself through the operator $\hat{O}$.

Formalization via P3.1 [4]: the lifetime of a configuration $T(C) = T_0/(1-S)$. For $S < 1$ the configuration is *unstable*: it exists for a finite time $T(C)$, after which it is replaced by the next one. The larger $|\mathcal{H}|$ (the more "contenders"), the stronger the pressure on the actualized configuration.

In the language of the φ-torus: the pressure manifests as a *shift along the ϕ-cycle*. Each θ-revolution actualizes a configuration, but the gap $\delta = \pi - 3$ "displaces" it from its original position — because the next configuration *does not coincide* with the previous one (gap $\neq 0$). The torus does not "inflate" — the point *advances* along its surface, and the area of the *covered* surface grows:

$$\mathcal{A}(n) = n \cdot 2\pi r \cdot \delta = 2\pi r n(\pi - 3) \tag{III.2}$$

III.3. Effective pressure force

The potentiality pressure per observation cycle:

$$\mathcal{F}_n = (\pi - 3)^2 \cdot \frac{|\mathcal{H}_{\text{accessible}}|}{|\mathcal{C}_n|} \tag{III.3}$$

Here $(\pi - 3)^2$ is the gap energy per revolution, and the ratio $|\mathcal{H}_{\text{accessible}}|/|\mathcal{C}_n|$ is a measure of the "overcrowdedness" of the potential field relative to the actualized configuration. Since $|\mathcal{H}| = \infty$ and $|\mathcal{C}| < \infty$:

$$\mathcal{F}_n \to \infty \quad \text{formally} \tag{III.4}$$

However, the operator $\hat{O}$ sees not the entire field $\mathcal{H}$ but only the states accessible from its dimensionality d (by D-Prot [4]). The number of accessible states is finite (though large), and the effective force is:

$$\mathcal{F}_{\text{eff}}(d) = (\pi - 3)^2 \cdot \Sigma(d) \cdot (1 - S)^{-1} \tag{III.5}$$

where $\Sigma(d) = (1 - q^{d+1})/(1 - q)$ is the spiral series sum [9], $(1 - S)^{-1}$ is the medium coherence factor [9, section IV].

III.4. Why the pressure never vanishes

By Ashby's law of requisite variety [10]: to fully control a system with n states, the controller must have at least n states. An observer with dimensionality d possesses a finite number of configurations. The field $\mathcal{H}$ is infinite. Therefore, $S = 1$ (full coherence, in which *all* potential states are actualized) is structurally unattainable [4, postulate P1.2]:

$$S < 1 \quad \text{always} \tag{III.6}$$

From (III.5) and (III.6): $\mathcal{F}_{\text{eff}} > 0$ always. The potentiality pressure *never* vanishes, because unrealized states always exist.

IV. SCALE FACTOR OF THE φ-TORUS

IV.1. Effective radius

A classical torus has fixed radii R and r. The ODTOE φ-torus *does not possess* a fixed radius in this sense. The effective radius of the configuration depends on the number of completed observational cycles.

After n cycles (θ-revolutions) at level d, the trajectory covers an area $\mathcal{A}(n)$ on the torus surface. The effective scale of actualized reality:

$$R_{\text{eff}}(n, d) = R_0 \cdot \varphi^d \cdot a(n) \tag{IV.1}$$

where $a(n)$ is the scale factor determined by gap accumulation.

IV.2. Derivation of the scale factor

Each θ-revolution shifts the point by $\delta = \pi - 3$ along ϕ. This shift *increases* the effective scale of the configuration by the fraction $\varepsilon/(2\pi\varphi)$, where $\varepsilon = (\pi - 3)^2$ is the gap energy, 2π is the length of a full θ-revolution, φ is the scale of the ϕ-cycle. Justification: the gap ε acts against the background of the full revolution 2π and is scaled through φ (the ratio of the torus radii), yielding the relative scale increment:

$$\frac{\Delta R}{R} = \frac{(\pi - 3)^2}{2\pi\varphi} = 0.001972031888168114672411398616 68\ldots \tag{IV.2}$$

The scale factor after n cycles:

$$\boxed{a(n) = \left(1 + \frac{(\pi - 3)^2}{2\pi\varphi}\right)^n} \tag{IV.3}$$

The numerical value of the expansion parameter:

$$H_{\text{ODTOE}} \equiv \frac{(\pi-3)^2}{2\pi\varphi} = 0.00197203188816811467241139861668 \tag{IV.4}$$

This is a *dimensionless* analogue of the Hubble parameter: the relative scale increment per observation cycle.

IV.3. Exponential growth

For $n \gg 1$:

$$a(n) \approx e^{nH_{\text{ODTOE}}} = e^{n(\pi-3)^2/(2\pi\varphi)} \tag{IV.5}$$

The expansion is *exponential*: the scale grows as the exponential of the number of observational cycles. This is consistent with the observed accelerated expansion of the Universe (de Sitter phase).

IV.4. Acceleration of expansion

First derivative (expansion rate):

$$\dot{a}(n) = H_{\text{ODTOE}} \cdot a(n) > 0 \tag{IV.6}$$

Second derivative (acceleration):

$$\ddot{a}(n) = H^2_{\text{ODTOE}} \cdot a(n) = \frac{(\pi-3)^4}{4\pi^2\varphi^2} \cdot a(n) > 0 \tag{IV.7}$$

The acceleration is strictly positive because $(\pi-3)^4 > 0$ (the square of a positive number). Accelerated expansion is not a free parameter but a *consequence* of the fact that $\pi \neq 3$.

$$\frac{(\pi-3)^4}{4\pi^2\varphi^2} = 0.00000388890976795189953370\ldots \tag{IV.8}$$

IV.5. Number of cycles for doubling the scale

$$n_{2\times} = \frac{\ln 2}{H_{\text{ODTOE}}} = \frac{\ln 2}{\ln(1+(\pi-3)^2/(2\pi\varphi))} \tag{IV.9}$$

Numerical computation:

$$n_{2\times} = \frac{0.69314\ldots}{0.00197008\ldots} = 351.84\ldots \tag{IV.10}$$

The scale doubles every ≈ 352 observational cycles (exact value: $351.84\ldots$).

V. A TORUS WITHOUT A FIXED RADIUS

V.1. Static and dynamic torus

Classical torus (Clifford, 1873): $R = \text{const}$, $r = \text{const}$. The geometry is fixed once and for all.

ODTOE φ-torus: $R/r = \varphi = \text{const}$ (the ratio is fixed by the KAM theorem [12, 13, 14]), but the *absolute* values of R and r depend on the number of completed cycles:

$$R(n,d) = R_0 \cdot \varphi^d \cdot a(n), \quad r(n,d) = r_0 \cdot \varphi^d \cdot a(n) \tag{V.1}$$

The ratio $R/r = R_0/r_0 = \varphi$ is preserved at every step. KAM stability is not violated. The torus *scales* while preserving its proportions.

V.2. Mechanism: potentiality pressure

Why does the scale grow? Because the field $\mathcal{H}$ "presses" on the configuration $\mathcal{C}$:

(a) Each cycle Φ actualizes a configuration C_n from $\mathcal{H}$.

(b) The configuration C_n *does not coincide* with C_{n-1}: the gap $\delta = \pi - 3 \neq 0$ guarantees that each new configuration *differs* from the previous one. The transcendence of π guarantees that the difference *never vanishes*.

(c) The new configuration C_n occupies a *new* region on the torus surface (one that was not previously covered by the trajectory).

(d) The totality of covered regions $\{C_0, C_1, \ldots, C_n\}$ constitutes the *actualized reality* at step n. Its effective scale grows as $a(n)$.

(e) Unrealized states from $\mathcal{H}$ continue to "press" at the next step, because $S < 1$ (Ashby).

V.3. Analogy

Imagine a sheet of paper ($\mathcal{C}$) lying on the ocean floor ($\mathcal{H}$). Water pressure from all sides *unfolds* the sheet, preventing it from collapsing. The deeper it lies (the larger $|\mathcal{H}|$), the stronger the pressure. The sheet does not "inflate" — it *unfolds*, covering an ever-larger area of the torus surface.

The "radius" of the torus does not grow as the physical inflation of a material object. What grows is the *effective scale of the actualized configuration* — the area of the toroidal surface "covered" by the observation trajectory.

V.4. Comparison with classical expansion

In standard cosmology, expansion is described by the Friedmann–Lemaitre–Robertson–Walker (FLRW) metric, where the scale factor $a(t)$ determines distances between comoving observers. The Friedmann equations govern the dynamics of $a(t)$ through energy density and pressure.

In ODTOE, the scale factor $a(n)$ (IV.3) describes not distances between points in a metric but the *volume of the actualized state space*. However, for an observer with dimensionality $d = 3$ (spatial three-dimensionality), the growth of $a(n)$ projects as an increase in spatial scales — which is observed as cosmological expansion.

The key distinction: in ΛCDM, expansion is described but not explained. The cosmological constant Λ is a free parameter. In ODTOE, expansion is *derived* from three structural elements: the transcendence of π (non-closure), the infinity of $\mathcal{H}$ (pressure), and the positivity of $(\pi - 3)^4$ (acceleration).

De Sitter expansion [23] — a special case of FLRW with $\Lambda > 0$ and no matter — is the closest analogue of ODTOE expansion at late stages ($n \gg 1$), when the scale factor grows exponentially. Hubble's observational data [24] and subsequent measurements confirm the transition of the Universe to the de Sitter phase.

VI. CONNECTION TO OBSERVED COSMOLOGY

VI.1. Dark energy = R-sector pressure

According to [6]: dark energy constitutes $\Omega_\Lambda = \varphi^2/(\varphi^2 + 1 + Z) = 68.86\%$ (Planck 2018: $68.47 \pm 0.73\%$, discrepancy 0.54σ). Physical mechanism: the R-sector of the φ-torus (major radius) carries the *potentiality pressure*. Rotation along ϕ (transition between levels d) scales as $R^2 = \varphi^2$, and it is precisely this sector that is responsible for the accelerated expansion.

Through the formalism of the present work: Ω_Λ is the fraction of total gravitational inertia attributable to the pressure of unrealized states. It is determined by the *geometry* of the torus (φ^2), not by the fitting parameter Λ.

VI.2. The cosmological constant problem

The standard problem: quantum field theory predicts $\rho_{\text{vac}} \sim m_P^4/(\hbar^3 c^3) \sim 10^{113}$ J/m^3, while the observed value is $\rho_\Lambda \sim 10^{-9}$ J/m^3. The discrepancy is $\sim 10^{122}$.

ODTOE answer: this is not a "problem" but a *property*. $|\mathcal{H}| = \infty$, while $|\mathcal{C}| < \infty$. The ratio $|\mathcal{H}|/|\mathcal{C}| \to \infty$. But an observer with dimensionality d sees not the entire $\mathcal{H}$ but only the $\Sigma(d)$-fraction — finite, determined by the recursion depth [9]. The observed "dark energy" $= (\pi - 3)^2 \cdot \Sigma(d)/(2\pi\varphi)$ — a finite number determined by the architecture of observation, not by vacuum fluctuations.

VI.3. Dark energy and de Sitter expansion

The scale factor (IV.3) for $n \gg 1$ gives:

$$a(t) \sim e^{H_{\text{ODTOE}} \cdot t/\tau_0} \tag{VI.1}$$

where t is physical time, τ_0 is the duration of one observational cycle. This is de Sitter expansion with a Hubble parameter:

$$H = \frac{H_{\text{ODTOE}}}{\tau_0} = \frac{(\pi - 3)^2}{2\pi\varphi\tau_0} \tag{VI.2}$$

The numerical agreement with the observed Hubble parameter ($H_0 \approx 70$ km/s/Mpc) is determined by τ_0 — the duration of the elementary observational cycle at level $d = 3$.

VI.4. Estimate of τ_0 from observations

From (VI.2) and the observed value $H_0 = 67.4$ km/s/Mpc [7]:

$$\tau_0 = \frac{H_{\text{ODTOE}}}{H_0} = \frac{0.00197203\ldots}{2.184 \times 10^{-18}\ \text{s}^{-1}} \approx 9.03 \times 10^{14}\ \text{s} \approx 2.86 \times 10^{7}\ \text{yr} \tag{VI.3}$$

The order $\tau_0 \sim 10^7$ yr is a characteristic time of a macroscopic observational cycle. This is consistent with the view that the cosmological scale factor is determined by *large-scale* observation dynamics rather than microscopic processes. For the quantum level ($d \gg 3$), the scale τ_0 will be different, determined by the decoherence time at the corresponding level.

Remark. All dimensionless results (H_{ODTOE}, Ω_Λ, Ω_{DM}, Ω_b, $n_{2\times}$) are obtained without free parameters — they are fully determined by π and φ. However, the transition to dimensional quantities requires τ_0, which in this work is determined through the observed Hubble parameter H_0. Epistemically, this is analogous to fitting Λ in ΛCDM: one free dimensional parameter. Deriving τ_0 from first principles is an open problem.

VI.5. Compatibility with the turbulent cascade picture

The scaling $R \propto \varphi^d$ (formula IV.1) is reminiscent of the Kolmogorov cascade [22] in turbulence: energy is transferred from scale d to scale $d+1$ with a constant scale ratio. In ODTOE, this ratio is fixed at the golden ratio φ rather than being a fitting parameter. The analogy with the turbulent cascade emphasizes that the expansion of the φ-torus is not a static inflation but a *dynamic actualization cascade*, transferring information (and scale) from level to level.

VII. HIERARCHY OF ETERNITY ARGUMENTS

The eternity of expansion is ensured not by one but by *four* complementary arguments from different areas of mathematics and theoretical physics.

VII.1. Argument 1: Transcendence of π (Lindemann's theorem)

The gap $\delta = \pi - 3$ is transcendental $\Rightarrow$ $N\delta \neq 2\pi k$ for any integers N, k $\Rightarrow$ the trajectory does not close $\Rightarrow$ expansion is eternal. (Section II.)

VII.2. Argument 2: Unattainability of $S = 1$ (Ashby's law)

$S < 1$ always $\Rightarrow$ $(1 - S)^{-1} > 1$ always $\Rightarrow$ potentiality pressure $\mathcal{F}_{\text{eff}} > 0$ always $\Rightarrow$ expansion is eternal. (Section III.)

VII.3. Argument 3: Infinity of $\mathcal{H}$

$|\mathcal{H}| = \infty$, $|\mathcal{C}| < \infty$ $\Rightarrow$ unrealized states always exist $\Rightarrow$ pressure does not vanish $\Rightarrow$ expansion is eternal. (Axiom A [4].)

VII.4. Argument 4: Positivity of $(\pi - 3)^4$

$\ddot{a} = H^2 a = [(\pi - 3)^4/(4\pi^2\varphi^2)] \cdot a > 0$ $\Rightarrow$ expansion is accelerated $\Rightarrow$ the expansion rate *grows* $\Rightarrow$ expansion cannot halt. (Section IV.)

Four arguments from four complementary sources: number theory (Lindemann), cybernetics (Ashby), ODTOE axiomatics (infinity of $\mathcal{H}$), differential calculus ($\ddot{a} > 0$). All four presuppose the toroidal ODTOE model; abandoning it removes the physical interpretation while preserving mathematical correctness.

VII.5. Remark on falsifiability

Each of the four arguments, taken separately, relies on a premise that *in principle* can be challenged:

(1) Lindemann's argument is irrefutable within mathematics — the transcendence of π is proved. However, one can challenge the *identification* of the torus traversal angle with π (i.e. the geometry of the model).

(2) Ashby's argument can be challenged if one admits an observer with infinite dimensionality ($d = \infty$), for which $S = 1$ is attainable. However, this contradicts D-Prot [4].

(3) The argument of the infinity of $\mathcal{H}$ can be challenged if one admits a finite potential field. This contradicts axiom (A) [4].

(4) The argument $\ddot{a} > 0$ depends on formula (IV.3) — which can be verified by comparison with data.

Thus, falsifying the eternity of expansion in ODTOE requires either abandoning the toroidal model or modifying the axioms — which is the standard procedure of scientific criticism.

VIII. DEMARCATION

Statement	Status	Source
π is transcendental	**Proved** (1882)	Lindemann's theorem [8]
$\delta = \pi - 3$ is transcendental	**Follows** from [8]	Algebra: difference of transc. and rat.
Trajectory on φ-torus does not close	**Follows** from transcendence of δ	Theorem 1 (Section II)
Expansion is eternal	**Follows** from non-closure	Four compl. arguments
$\mathcal{F}_{\text{eff}} > 0$ always	**Follows** from $S < 1$	Ashby's law [10] + P3.1 [4]
$a(n) = (1 + \varepsilon/(2\pi\varphi))^n$	**Derived** from spiral gap	Formulas (IV.2)–(IV.3)
$\ddot{a} > 0$ (accelerated expansion)	**Follows** from $(\pi - 3)^4 > 0$	Formula (IV.7)
$\Omega_\Lambda = 68.86\%$	Agrees with Planck (0.54σ)	[6]
φ-torus has no fixed R	ODTOE interpretation	Section V
Dark energy = pressure of $\mathcal{H}$ on $\mathcal{C}$	ODTOE interpretation	[4, 6]
$n_{2\times} \approx 351.84$ cycles	**Computed**	Formula (IV.10)
$\tau_0 \sim 10^7$ yr (estimate)	Follows from H_0 [7] and (VI.2)	Formula (VI.3)
Uniform filling of φ-torus	**Follows** from Weyl's theorem [25]	Section II.2a

Remark. All statements marked "**Proved**" or "**Follows**" rest on mathematical theorems (Lindemann, Weyl, Banach [21]) and the ODTOE axiomatics [4]. Statements marked "ODTOE interpretation" are consequences of the model and are subject to empirical verification. Statements marked "Agrees with Planck" represent quantitative predictions already consistent with data [7] within 1σ.

Note also that the *computed* values ($a(n)$, H_{ODTOE}, $n_{2\times}$) contain no free parameters — they are fully determined by the fundamental mathematical constants π and φ. The only parameter requiring independent determination is τ_0 (the observational cycle duration), which connects the dimensionless scale factor to physical time.

VIII-bis. COSMOLOGICAL FRACTIONS FROM TOROIDAL ARCHITECTURE

The toroidal expansion model developed in this paper admits a direct consequence for the cosmological composition of the Universe [6]. The φ-torus with $R/r = \varphi$ possesses three topological sectors. Below we present the full derivation of cosmological fractions from π and φ.

VIII-bis.1. Gravitational inertia of sectors

Each degree of freedom of the φ-torus contributes to the total gravitational inertia. For rotational motion the effective mass is proportional to the square of the characteristic radius:

$$M_{\text{eff}} \propto r_{\text{eff}}^2 \tag{VIII-bis.1}$$

Inter-level sector (rotation along the major radius R): transition between dimensionality levels d. Effective mass $\propto R^2 = \varphi^2 r^2$. In ODTOE: pressure of the field of potential states $\mathcal{H}$ on the configuration space $\mathcal{C}$. Identified with **dark energy** (Ω_Λ).

Intra-level sector (rotation along the minor radius r): phase dynamics within a single level d. Effective mass $\propto r^2 = 1$ (in units of r). In ODTOE: coherent configurations at levels $d > d_{\text{our}}$, invisible by D-Prot but gravitating by P5 [4]. Identified with **dark matter** (Ω_{DM}).

Ratio of gravitational weights:

$$\frac{I_R}{I_r} = \frac{R^2}{r^2} = \varphi^2 = 2.61803398... \tag{VIII-bis.2}$$

VIII-bis.2. Gap sector: derivation of Z from π and φ

Each revolution along the minor radius does not close: path length $= \pi$, minimum closed $= 3$ (ternary architecture [16]). First-turn gap: $\delta_1 = \pi - 3$. Each subsequent turn is scaled by φ (step between turns on the torus). The k-th order gap: $(\pi - 3)^k \cdot \varphi^{k-1}$. Summing the infinite geometric series (converges since $(\pi - 3)\varphi = 0.2291... < 1$):

$$Z = \sum_{k=1}^{\infty} (\pi - 3)^k \cdot \varphi^{k-1} = \frac{\pi - 3}{1 - (\pi - 3)\varphi} = \frac{0.14159...}{0.77090...} = 0.18367... \tag{VIII-bis.3}$$

Physical meaning: visible matter = the sum of all spiral gaps generated by non-closure of the observation loop. Photons, atoms, stars, observers are all born in this gap [14].

VIII-bis.3. Normalized fractions

Total weight:

$$\Sigma = \varphi^2 + 1 + Z = 2.61803 + 1 + 0.18367 = 3.80171 \qquad \text{(VIII-bis.4)}$$

Normalized fractions:

$$\Omega_\Lambda = \frac{\varphi^2}{\Sigma} = \frac{2.61803}{3.80171} = 68.86\% \qquad \text{(VIII-bis.5)}$$

$$\Omega_{DM} = \frac{1}{\Sigma} = \frac{1}{3.80171} = 26.30\% \qquad \text{(VIII-bis.6)}$$

$$\Omega_b = \frac{Z}{\Sigma} = \frac{0.18367}{3.80171} = 4.83\% \qquad \text{(VIII-bis.7)}$$

Check: $68.86 + 26.30 + 4.83 = 100.00\%$.

VIII-bis.4. Comparison with Planck 2018

Component	ODTOE, %	Planck 2018 [7], %	Dev.	σ
Dark energy (Ω_Λ)	**68.86**	68.47 ± 0.73	$+0.39$	**0.54**
Dark matter (Ω_{DM})	**26.30**	26.60 ± 0.73	-0.30	**0.41**
Baryonic (Ω_b)	**4.83**	4.93 ± 0.06	-0.10	**1.64**

Dark energy and dark matter: *within* 1σ. Baryonic: *within* 2σ (1.64σ). A self-referential correction (by analogy with μ and α^{-1} [16]) improves the baryonic agreement to 1.24σ [6].

VIII-bis.5. Connection to expansion

In the limit $\pi \to 3$ the gap $Z \to 0$, and the ternary proportion degenerates into the binary one:

$$\lim_{\pi\to 3} \frac{\varphi^2}{\varphi^2 + 1 + Z} = \frac{\varphi^2}{\varphi^2 + 1} = \frac{\varphi}{1+\varphi} = 61.8\% \qquad \text{(VIII-bis.8)}$$

The binary φ-proportion 62/38 is observed in optimal biological regimes (systole/diastole, inhalation/exhalation) [12]. The inequality $\pi > 3$ is the reason why cosmological fractions *differ* from the "pure" φ-proportion and generate visible matter as a by-product of topological frustration. The Universe consists of $\sim 95\%$ "torus" ($\varphi^2 + 1$) and $\sim 5\%$ "gap" (Z): that which is born at each non-closure of the loop.

IX. CONCLUSION

IX.1. Results

First. Within the φ-torus model, the eternity of expansion is proved as a mathematical theorem: the transcendence of π (Lindemann's theorem, 1882) forbids closure of the trajectory on the φ-torus in any finite number of revolutions. The mathematical part holds with the same rigor as the impossibility of squaring the circle; the physical interpretation depends on the toroidal model.

Second. Potentiality pressure is formalized: the infinite field of unrealized states $\mathcal{H}$ exerts on the finite configuration $\mathcal{C}$ an effective force $\mathcal{F}_{\text{eff}} = (\pi - 3)^2 \cdot \Sigma(d) \cdot (1 - S)^{-1} > 0$. The pressure never vanishes thanks to the structural unattainability of $S = 1$ (Ashby's law).

Third. It is shown that the φ-torus does not possess a fixed radius: the scale factor $a(n) = (1 + (\pi - 3)^2/(2\pi\varphi))^n$ describes the exponential growth of the effective scale with the number of observational cycles, preserving the ratio $R/r = \varphi$ (KAM stability).

Fourth. Accelerated expansion ($\ddot{a} > 0$) is derived from $(\pi - 3)^4 > 0$ without the cosmological constant as a free parameter. Dark energy is interpreted as the pressure of the R-sector of the φ-torus [19, 20] ($\Omega_\Lambda = \varphi^2/(\varphi^2 + 1 + Z) = 68.86$%, Planck: 68.47%, discrepancy 0.54σ).

IX.2. One formula

$$\boxed{a(n) = \left(1 + \frac{(\pi - 3)^2}{2\pi\varphi}\right)^n, \quad \ddot{a} > 0, \quad \text{closure impossible (Lindemann, 1882)}}$$

The expansion of reality is eternal because π is transcendental. The expansion is accelerated because $(\pi - 3)^4 > 0$. The expansion is inexhaustible because $|\mathcal{H}| = \infty$ and $S < 1$ (Ashby). Three numbers — π, φ, $(\pi - 3)^2$ — and one theorem from 1882. The convergence of the loop Φ to the fixed point Ψ^* is ensured by the contraction mapping principle [21]; the self-referential equations for μ and α^{-1} [11] yield the same invariants (π, φ), confirming the structural unity of the formalism.

IX.3. Prospects

The following questions remain open:

(1) Derivation of τ_0 from first principles of ODTOE — the connection between the elementary observational cycle duration and the dimensionality d and coherence parameter S.

(2) Description of the transition from decelerated expansion (matter-dominated epoch) to accelerated expansion (de Sitter phase) in terms of changing effective observation dimensionality. In ΛCDM this transition occurs at $z \approx 0.7$; in ODTOE it should correspond to a critical value S_{cr} at which potentiality pressure begins to dominate over the material component.

(3) Phenomenology of fluctuations: the power spectrum of the cosmic microwave background and its connection to the discrete structure of the φ-torus (fractal scale correlations [26]).

(4) Connection of the scale factor $a(n)$ with the entropic characteristics of the configuration — a possible formalization of the arrow of time as the direction of growth of $a(n)$.

(5) Experimental verification: searching for discrete correlations in the CMB spectrum that would correspond to the spiral structure of the φ-torus. The characteristic angular scale of such correlations is determined by the ratio $\delta/(2\pi) = (\pi-3)/(2\pi) \approx 0.02254$, corresponding to multipoles $\ell \approx 1/0.02254 \approx 44$. Planck data [7] contain anomalies at low multipoles that may be related to toroidal topology.

(6) Formalization of the connection between potentiality pressure and gravity: if dark energy is the projection of the R-sector pressure of the φ-torus, then there should exist a formal equivalence between the Friedmann equations (for the de Sitter phase) and the discrete recursion $a(n+1) = (1 + H_{\mathrm{ODTOE}}) \cdot a(n)$.

ACKNOWLEDGEMENTS AND TOOLS

The author thanks anonymous reviewers for constructive comments that improved the exposition.

In the development of the ODTOE theory and all articles based on it, artificial intelligence tools were used: Claude Sonnet / Opus 4.6 Extended (Chat & Code) (Anthropic), ChatGPT 5.3 (OpenAI), Google Gemini (Google DeepMind). All substantive decisions, hypotheses, interpretations, and responsibility for them belong to the author.

CONFLICT OF INTEREST

The author declares no conflict of interest.

FUNDING

This work was carried out without external funding.

REFERENCES

[1] Riess A.G. et al. Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant // The Astronomical Journal. — 1998. — Vol. 116. — P. 1009–1038. DOI: 10.1086/300499.

[2]Perlmutter S. et al. Measurements of Ω and Λ from 42 High-Redshift Supernovae // The Astrophysical Journal. — 1999. — Vol. 517. — P. 565–586. DOI: 10.1086/307221.

[3]Weinberg S. The Cosmological Constant Problem // Reviews of Modern Physics. — 1989. — Vol. 61. — P. 1–23. DOI: 10.1103/RevModPhys.61.1.

[4]Pankratov A.S. Theory of Everything: Observer-Dependent (ODTOE) // Preprint. — 2025. — 47 p.

[5]Pankratov A.S. Toroidal Topology of Reality: Nested φ-Tori // Preprint. — 2026.

[6]Pankratov A.S. Cosmological Proportions from Toroidal Architecture // Preprint. — 2026.

[7]Planck Collaboration (Aghanim N. et al.) Planck 2018 results. VI. Cosmological parameters // Astronomy & Astrophysics. — 2020. — Vol. 641. — Art. A6. DOI: 10.1051/0004-6361/201833910.

[8]Lindemann F. Ueber die Zahl π // Mathematische Annalen. — 1882. — Bd. 20. — S. 213–225. DOI: 10.1007/BF01446522.

[9]Pankratov A.S. Planck's Constant from the Architecture of Observation // Preprint. — 2026.

[10]Ashby W.R. An Introduction to Cybernetics. — London: Chapman & Hall, 1956.

[11]Pankratov A.S. Two Fundamental Constants from First Principles: μ and α^{-1} // Preprint. — 2026.

[12]Kolmogorov A.N. On the preservation of conditionally periodic motions // Doklady Akademii Nauk SSSR. — 1954. — Vol. 98. — P. 527–530.

[13]Arnold V.I. Small denominators and problems of stability of motion // Uspekhi Matematicheskikh Nauk. — 1963. — Vol. 18(6). — P. 91–192.

[14]Moser J. On Invariant Curves of Area-Preserving Mappings of an Annulus // Nachr. Akad. Wiss. Gottingen, Math.-Phys. Kl. II. — 1962. — P. 1–20.

[15]Khinchin A.Ya. Continued Fractions. — Chicago: University of Chicago Press, 1964.

[16]Pankratov A.S. The Number π as a Structural Invariant of Self-Consistent Observation // Preprint. — 2025.

[17]Niven I. Irrational Numbers. — Mathematical Association of America, 1956.

[18]Baker A. Transcendental Number Theory. — Cambridge University Press, 1975.

[19]Pankratov A.S. $\mathbb{Z}_2$-Bundle over the φ-Torus: Spinor Architecture of Fundamental Constants // Preprint. — 2026.

[20]Pankratov A.S. Ether, Vacuum and the Potentiality Field: from Newton to ODTOE // Preprint. — 2026.

[21]Banach S. Sur les operations dans les ensembles abstraits et leur application aux equations integrales // Fundamenta Mathematicae. — 1922. — Vol. 3. — P. 133–181.

[22]Kolmogorov A.N. The local structure of turbulence in incompressible viscous fluid for very large Reynolds numbers // Proceedings of the USSR Academy of Sciences. — 1941. — Vol. 30. — P. 299–303.

[23]de Sitter W. Einstein's theory of gravitation and its astronomical consequences. Third paper // Monthly Notices of the Royal Astronomical Society. — 1917. — Vol. 78. — P. 3–28.

[24]Hubble E. A Relation between Distance and Radial Velocity among Extra-Galactic Nebulae // Proceedings of the National Academy of Sciences. — 1929. — Vol. 15. — P. 168–173.

[25]Weyl H. Über die Gleichverteilung von Zahlen mod. Eins // Mathematische Annalen. — 1916. — Bd. 77. — S. 313–352.

[26]Pankratov A.S. The Golden Ratio φ as an Invariant of Fractality, Self-Similarity and Recursion in ODTOE // Preprint. — 2026.