FULL DERIVATION OF EINSTEIN EQUATIONS FROM ODTOE: SYNTHESIS OF THE FOUR-ARTICLE PROGRAMME

(Полный вывод уравнений Эйнштейна из ODTOE: синтез четырёх-статейной программы)

Programme A→B→C→XL: tensor structure, source, closure; Programme Completion Theorem T0

Pankratov Anton Sergeevich

Панкратов Антон Сергеевич

Independent researcher, Kazan, Russia
E-mail: anton.s.pankratov@gmail.com
ORCID: 0009-0002-4870-2995

UDC 530.12 + 530.145 + 514.764.2 + 524.85

ABSTRACT

This paper synthesizes the full derivation of the Einstein equations from ODTOE, carried out in the three-stage programme §XIV.3 of [13] (*ODTOE_gravity_causal_structure*, the historically first work formalizing the causal layer as stage 1 of the derivation). The programme is realized by three independent, sequentially dependent articles: § A — tensor structure [14] (metric $g_{\mu\nu}$ as observer-correlator, covariant derivative ∇_μ as Φ-iteration commutator, Riemann tensor $R^\rho{}_{\sigma\mu\nu}$ via non-commutativity of SYNC operations, theorems A.T1–A.T5, Schwarzschild and Kerr solutions); § B — tensor source [15] (observer action $S_{\text{obs}} = \int B^2(1-\sigma)\Lambda\sqrt{-g}\,d^4x$, SYNC projector $P_{O,\text{SYNC}}$, lemma L7 on idempotency $P^2_{O,\text{SYNC}} = P_{O,\text{SYNC}}$, lemma L8 on conservation $\nabla_\mu T^{\mu\nu} = 0$, closed form $\chi_\Lambda(S^*) \approx 0.082201$ giving $\Omega_\Lambda \approx 0.688647$ in agreement with Planck 2018 within 0.05σ); § C — closure [16] (theorem C.T1 on Φ-self-consistency $G_{\mu\nu} + \Lambda g_{\mu\nu} = (8\pi G/c^4)T_{\mu\nu} \Leftrightarrow \Phi_C(g,T) = (g,T)$, theorem C.T2 on the dual-path Bianchi identity $\nabla_\mu G^{\mu\nu} = 0$, theorem C.T3 — ODTOE analog of the Hawking–Penrose singularity theorem). The present XL paper formulates and grounds the programme completion theorem T0: the combined results of A+B+C suffice to derive the full dynamical Einstein equation $G_{\mu\nu} + \Lambda g_{\mu\nu} = (8\pi G/c^4)T_{\mu\nu}$ from ODTOE primitives; the standard solutions (Schwarzschild, Kerr, FLRW) are recovered as exact ODTOE constructions, not as ansätze. The programme §XIV.3, declared open in [13], is thereby semantically closed; the original disclaimer formulation of §I in [13] (lines 117–120) is a historical artifact reflecting the state prior to completion of the present synthetic work. The paper closes the four-article programme cycle and fixes the programme completion theorem T0 for subsequent works of the corpus.

Keywords: ODTOE, Einstein equation, Φ-self-consistency, Banach theorem, Bianchi identity, Noether theorem, Diff(M^4), singularity theorem, Schwarzschild, Kerr, FLRW, $\chi_\Lambda(S^*)$, Ω_Λ, programme §XIV.3, theorem T0, programme completion, synthesis

АННОТАЦИЯ

В настоящей работе синтезирован полный вывод уравнений Эйнштейна из ODTOE, выполненный в трёхэтапной программе §XIV.3 из [13] (*ODTOE_gravity_causal_structure*, исторически первая работа, формализующая причинный слой как первый этап деривации). Программа реализована тремя независимыми, последовательно опирающимися статьями: § A — тензорная структура [14] (метрика $g_{\mu\nu}$ как observer-correlator, ковариантная производная ∇_μ как Φ-итерационный коммутатор, тензор Римана $R^\rho{}_{\sigma\mu\nu}$ через некоммутативность SYNC-операций, теоремы A.T1–A.T5, решения Шварцшильда и Керра); § B — тензорный источник [15] (действие наблюдателя S_{obs}, SYNC-проектор $P_{O,\text{SYNC}}$, леммы L7 и L8, замкнутая форма $\chi_\Lambda(S^*) \approx 0{,}082201$, дающая $\Omega_\Lambda \approx 0{,}688647$ в согласии с Planck 2018 в пределах $0{,}05\sigma$); § C — замыкание [16] (теоремы C.T1, C.T2, C.T3). Настоящая статья XL формулирует и обосновывает теорему T0 о завершении программы. Программа §XIV.3, заявленная в [13] как открытая, тем самым семантически замкнута.

Ключевые слова: ODTOE, уравнение Эйнштейна, Φ-самосогласованность, теорема Банаха, тождество Бианки, теорема Нётер, Diff(M^4), теорема о сингулярностях, Шварцшильд, Керр, FLRW, $\chi_\Lambda(S^*)$, Ω_Λ, программа §XIV.3, теорема T0, замыкание программы, синтез.

I. INTRODUCTION: PURPOSE OF XL AND STATUS OF THE ORIGINAL DISCLAIMER

I.1. Purpose of the XL paper

The purpose of the present work is to synthesize the four-article programme aiming at the full derivation of the Einstein equation

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{1.1}$$

from ODTOE primitives, and to document its completion. The programme is realized sequentially: the causal structure [13], tensor structure [14], tensor source [15], and closure (the field equation as Φ-self-consistency with the dual-path Bianchi identity and the ODTOE analog of the singularity theorem) [16]. The present XL paper introduces no new derivation steps: it cites already fixed results and formulates a general structural statement — the programme completion theorem T0 — performing the function of the keystone of the arch.

Epistemic status. The work is *synthetic*: T0 is not a new theorem but a structural statement combining theorems A.T1–A.T5, lemmas L7–L8, and theorems C.T1–C.T3 into a single chain § A→ § B→ § C→ § XL. The proof of T0 is the derivation chain itself; the recaps in §II–§IV contain brief citations without re-derivation.

I.2. Status of the original disclaimer in [13]

The work [13] *ODTOE_gravity_causal_structure* was written as the first stage of the derivation; in its §I an epistemic disclaimer was placed (lines 117–120 of the source) fixing that the article *does not claim* a full derivation of the Einstein equations from ODTOE and formalizes only the causal layer necessary for such a derivation. The full derivation programme was formulated in [13] §XIV.3 as open, with three structural requirements: (1) tensor structure of $g_{\mu\nu}$ from microSYNC; (2) $T_{\mu\nu}$ as functional derivative of the B-functional; (3) Bianchi identities from Φ-self-consistency. As of the completion of the present work, all three requirements are met: item (1) is realized in [14], item (2) — in [15], item (3) — in [16].

The semantic status of the disclaimer [13] §I (lines 117–120) is thereby retired by the present synthesis: programme §XIV.3 is performed. However, the work [13] itself remains the canonical formalization of the causal layer as stage 1 of the derivation; its disclaimer formulation and §XIV.3 "Open programme" historically fix the state prior to completion of the programme. A reader consulting [13] should interpret these formulations in the context of the completed programme documented in the present work. See the detailed discussion in §X.

I.3. Structure of the exposition

§II–§IV contain recaps of articles A, B, C, one page each, with slug-citation of the central results. §V is the synthesis: visualization of the chain ODTOE primitives $\to$A$\to$B$\to$C$\to$ field equation, with two anchor formulas highlighted. §VI–§VIII contain the exact solutions (Schwarzschild, Kerr, FLRW) as ODTOE fixed points of Φ_C. §IX formulates and grounds the programme completion theorem T0. §X — "Relation to [13]" — gives a detailed account of the disclaimer status. §XI discusses the post-Einstein outlook and future programmes. §XII is the conclusion. Then follow the sections of acknowledgements, conflict of interest, and funding (per L-33), and after them — the bibliography.

II. RECAP A: TENSOR STRUCTURE (1 PAGE)

Article [A] = [14] *ODTOE_gravity_tensor_structure* closed stage 1 of programme [13] §XIV.3. Six structural results are fixed:

- **Metric $g_{\mu\nu}$ as observer-correlator** [14] formula (F1):

$$g_{\mu\nu}(C; O) = \langle \partial_\mu \Phi,\, \partial_\nu \Phi \rangle_{O,C} \tag{A.F1}$$

  where $\Phi = \iota \circ \hat{O}$ is the self-observation map, $\langle \cdot, \cdot \rangle_{O,C}$ is the SYNC-induced inner product in $\mathcal{H}$. Symmetry and non-degeneracy in the macro-limit recover a pseudo-Riemannian metric with signature $(-,+,+,+)$ in the MTW convention [7].

- **Covariant derivative ∇_μ as Φ-iteration commutator** [14] formula (F3):

$$\nabla_\mu V^\nu = \lim_{\Delta x \to 0} \frac{1}{\Delta x}\left[\Phi^{(\mu)}_{\Delta x}\, V^\nu - V^\nu(x + \Delta x\, \hat{e}_\mu)\right] \tag{A.F3}$$

  with recovery of the Levi-Civita Christoffel symbols by Theorem A.T1.

- **Riemann curvature tensor $R^\rho{}_{\sigma\mu\nu}$ as a measure of non-commutativity of SYNC operations** [14] formula (F5): $R^\rho{}_{\sigma\mu\nu}V^\sigma = [\nabla_\mu, \nabla_\nu]V^\rho$, the coordinate form (F6) coincides with MTW [7] (8.45) and Wald [18] (3.2.3).

- **Einstein tensor** $G_{\mu\nu} = R_{\mu\nu} - (1/2)g_{\mu\nu}R$ [14] formula (F9), the kinematic Bianchi identity $\nabla_\mu G^{\mu\nu} = 0$ as a purely geometric consequence of metric smoothness (Theorem A.T3).

- **Inertial scalar potential** Π_I — a unified notation for the scalar formalizing §V.1 of [13]; the legacy symbol Φ_I from [12] is replaced by Π_I (see [14] §II.2 and [15] §II.1, footnote).

- **Schwarzschild solution (Theorem A.T4) and Kerr solution (Theorem A.T5)** as exact ODTOE constructions; the Kerr solution in Boyer–Lindquist coordinates [8] is derived as a spherically-axial ansatz with a vortex SYNC component induced by the angular momentum of the source. A 50-digit numerical demonstration reproduces the perihelion shift of Mercury $\Delta\phi = 42.99$ arcsec/century and the position of the equatorial ergosphere $r_E^{\text{eq}} = 2M$ [14] §IX.

These six contracts are used in the present XL paper without re-derivation. Everything required from § A in the synthesis §IX (theorem T0) is already fixed: $g_{\mu\nu}$, ∇_μ, $R^\rho{}_{\sigma\mu\nu}$, $G_{\mu\nu}$, Π_I, and the exact solutions are structural inputs for § B and § C.

III. RECAP B: TENSOR SOURCE $T_{\mu\nu}$ AND CLOSED FORM $\chi_\Lambda(S^*)$ (1 PAGE)

Article [B] = [15] *ODTOE_gravity_T_munu_projector* closed stage 2 of programme [13] §XIV.3. Six structural results are fixed:

- **Observer action** S_{obs} [15] formula (F4):

$$S_{\text{obs}}[g, B, \sigma, \Lambda] = \int_{\mathcal{M}^4} B(O,C)^2\,(1 - \sigma(O,C))\,\Lambda(O,C)\,\sqrt{-g}\,d^4x \tag{B.F4}$$

  with integrand density $\mathcal{L}_{\text{obs}} = B^2(1-\sigma)\Lambda$ — the local density of observer coherence.

- **SYNC projector** $P_{O,\text{SYNC}} : \mathcal{H} \to \mathcal{C}$ — orthogonal projection onto the closed Φ-invariant subspace $\mathcal{C} = \text{Fix}(\Phi) \cap \mathcal{H}_{\text{coh}}$ [15] formula (F8); existence and uniqueness are secured by the orthogonal projection theorem in Hilbert space [1] Thm II.3.

- **Stress-energy tensor $T_{\mu\nu}$ via variational derivative** [15] formulae (F15)–(F16):

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\,\mathcal{L}_{\text{obs}})}{\delta g^{\mu\nu}} = 2B^2(1-\sigma)\Lambda\,(P_{O,\text{SYNC}})_{\mu\nu} - g_{\mu\nu}B^2(1-\sigma)\Lambda \quad \text{(B.F15)}$$

- **Lemma L7 on idempotency** $P^2_{O,\text{SYNC}} = P_{O,\text{SYNC}}$ — proved in [15] §V via four sub-lemmas (L7.1 closedness, L7.2 linearity, L7.3 well-definedness, L7.4 self-adjointness) with explicit anti-circularity audit: the Bianchi identity and the Einstein equation are not used.

- **Lemma L8 on the conservation law** $\nabla_\mu T^{\mu\nu} = 0$ — proved in [15] §VII via the covariant derivative fixed in [14] §IV.1 (formula A.F3) and the idempotency L7. Conservation is a consequence of Φ-self-consistency, not an axiom.

- **Closed form of the cosmological constant** $\chi_\Lambda(S^*)$ [15] formula (F23):

$$\chi_\Lambda(S^*) = \frac{3\varphi^2}{8\pi(\varphi^2 + 1 + Z(S^*))}, \qquad Z(S^*) = \frac{\pi - 3}{1 - (\pi - 3)\varphi} \quad \text{(B.F23)}$$

  with substitution of 50-digit constants: $\chi_\Lambda(S^*) \approx 0.082201$ and $\Omega_\Lambda(S^*) \approx 0.688647$, which agrees with Planck 2018 [10] $\Omega_\Lambda = 0.6889 \pm 0.0056$ within 0.05σ *without fitting*. This closes the fitted form $\chi_\Lambda \simeq 8.2 \cdot 10^{-2}$ from [12] §XII.5. The value of global coherence $S^* \approx 0.169676$ used here is consistent with the independent derivation of the gravitational constant from ODTOE first principles under the structural hypothesis $C = B^2$ [26] §IV (the same S^* calibration), which secures the compatibility of the B-channel and the G-channel.

The six B-contracts enter the synthesis of theorem T0 in §IX as the tensor source: $T_{\mu\nu}$ from the B-functional, projector idempotency (L7), conservation (L8), closed form of Λ.

IV. RECAP C: PROGRAMME CLOSURE (1 PAGE)

Article [C] = [16] *ODTOE_einstein_derivation_complete* closed stage 3 of programme [13] §XIV.3. Three central theorems are fixed:

- **Theorem C.T1 (Φ-self-consistency)** — a pair $(g, T) \in C_{\text{contr}}$ satisfies the Einstein equation (1.1) iff (g, T) is a fixed point of the map $\Phi_C = \iota \circ \hat{O}$ on the Φ-invariant subspace of pairs:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \iff \Phi_C(g, T) = (g, T) \quad \text{(C.F11)}$$

  Existence and uniqueness modulo Diff(M^4) are secured by the Banach fixed-point theorem [6] for a contraction map. The contraction argument uses only geometric estimates and observer-action bounds and *does not assume* the Einstein equation — the anti-circularity audit is performed explicitly in [16] §VI.6.

- **Theorem C.T2 (dual-path Bianchi identity)** — the identity $\nabla_\mu G^{\mu\nu} = 0$ is established along two independent paths: Path 1 — kinematic via Theorem A.T3 of [14] (contraction of the second Bianchi identity on a smooth pseudo-Riemannian metric); Path 2 — dynamical via Noether's theorem [2] for S_{obs} under the action of Diff(M^4). Numerical verification on the Schwarzschild ground state in 50-digit `mpmath` arithmetic gives

$$\left|\nabla_\mu G^{\mu\nu}\right|_{\text{Path 1}} - \left|\nabla_\mu G^{\mu\nu}\right|_{\text{Path 2}} < 10^{-45} \tag{C.F9}$$

  The anti-circularity audit of both paths is performed in [16] §IV.4.

- **Theorem C.T3 (ODTOE analog of the singularity theorem)** — under the ODTOE energy condition (derived from L8 in [15] §VII), the trapped-configuration analog via the causal cone J^+_O of [13] §VI, and the ontological collapse condition $B \to 0$ of [17] §VII.3, there exists a Φ-iteration sequence of finite affine parameter terminating in the Fix(Φ) attractor with no successor in J^+_O. This is the structural analog of the Hawking–Penrose theorem [3, 4].

Honest status of § *C.* Theorem C.T3 in [16] §VII.5 is explicitly accompanied by an open task: the full topological formalization of the limit $B \to 0$ as a boundary point of Φ-iteration is left for future publication. This reservation is inherited in §IX of the present work. All three theorems C.T1–C.T3 are PROVED with the status "proved with explicit indication of open tasks."

The three C-theorems enter the synthesis of T0 in §IX as the programme closure: equivalence of the Einstein equation to Φ-self-consistency (C.T1), circuit-free proof of $G_{\mu\nu}$ conservation (C.T2), and structural extension of the singularity theorem in ODTOE (C.T3).

V. SYNTHESIS: COMPLETE DERIVATION CHAIN FROM ODTOE PRIMITIVES TO THE FIELD EQUATION

V.1. Structural diagram of the chain

The full chain of derivation from ODTOE primitives to the Einstein equation (1.1) is visualized as four sequential transitions:

$$\text{ODTOE primitives } (\mathcal{H}, \mathcal{C}, \hat{O}, B, I, S) \;\to\; \text{[A]} : g_{\mu\nu}, \nabla_\mu, R^\rho{}_{\sigma\mu\nu}, G_{\mu\nu} \;\to\; \text{[B]} : T_{\mu\nu}, \chi_\Lambda(S^*) \;\to\; \text{[C]} : \Phi_C\text{-sel} \tag{XL.F1}$$

In terms of structural operations:

- **Step 1 (from primitives to geometry).** The self-observation map $\Phi = \iota \circ \hat{O}$ on the configuration manifold $\mathcal{C}$ generates the observer-correlator $g_{\mu\nu}$ (formula A.F1), the Φ-iteration commutator defines ∇_μ (formula A.F3), the non-commutativity of SYNC along two directions defines the Riemann tensor and further $G_{\mu\nu}$ (Theorem A.T3 — kinematic Bianchi identity). The dimensional anchor A_0, securing the φ-invariant Planck constant $\hbar$ from toroidal geometry and observer coherence, is derived in [27] §V and fixes the action scale for all formulas of the chain.

- **Step 2 (from geometry to source).** The observer action $S_{\text{obs}} = \int B^2(1-\sigma)\Lambda\sqrt{-g}\,d^4x$ (formula B.F4) gives $T_{\mu\nu}$ as the functional derivative $\delta S_{\text{obs}}/\delta g^{\mu\nu}$ (formula B.F15) with PROVED idempotency of the SYNC projector (L7) and PROVED conservation law (L8). The closed form $\chi_\Lambda(S^*)$ (formula B.F23) gives the cosmological constant from the global coherence of the Universe $S^* \approx 0.169676$; the principle P5 of collective actualization, by virtue of which S^* is operationally meaningful precisely as the coherence of an observer cluster rather than a single world-line, is formalized in [25] §III.
- **Step 3 (closure).** The Φ-self-consistency condition on pairs (g, T) — Theorem C.T1 — establishes the equivalence of the Einstein equation (1.1) to the fixedness $\Phi_C(g, T) = (g, T)$. The dual-path Bianchi identity (C.T2) ensures the compatibility of $G_{\mu\nu}$ and $T_{\mu\nu}$ via L8 and Noether symmetry. The singularity theorem (C.T3) gives the ODTOE analog of the classical results [3, 4].
- **Step 4 (synthesis T0).** The combination of steps 1–3 gives the full derivation of (1.1) from ODTOE primitives; standard solutions are recovered as exact ODTOE constructions (see §VI–§VIII). The canonical form of the unified self-observation operator Φ and its treatment as a contraction map with Fix(Φ) as the universal attractor are presented in [24] §II–§III; this canonical form is precisely what is reused in C.T1 for the Einstein equation (1.1).

V.2. Anchor formula 1: Einstein equation as Φ-fixed point

The first anchor formula of the synthesis is the reformulation of (1.1) as a Φ-self-consistency condition (C.T1):

$$\boxed{G_{\mu\nu}[g] + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}[g, B, \sigma, \Lambda] \iff \Phi_C(g, T) = (g, T)} \tag{XL.F2}$$

where $\Phi_C = \iota \circ \hat{O}$ is the induced map on pairs $(g, T) \in C_{\text{contr}}$, $\hat{O} : g \mapsto T$ is the variational derivative from [15], and $\iota : T \mapsto g$ is the inverse map via uniqueness of the Einstein equation solution with given T (modulo Diff(M^4)).

V.3. Anchor formula 2: cosmological constant from Universe coherence

The second anchor formula of the synthesis is the closed form of the cosmological constant from the global coherence of the Universe (formula B.F23 with substitution of 50-digit constants):

$$\boxed{\chi_\Lambda(S^*) = \frac{3\varphi^2}{8\pi(\varphi^2 + 1 + Z(S^*))} \implies \Omega_\Lambda(S^*) = \frac{\varphi^2}{\varphi^2 + 1 + Z(S^*)} \approx 0.68864709\ldots} \tag{XL.F3}$$

with $Z(S^*) = (\pi - 3)/(1 - (\pi - 3)\varphi)$ and value $S^* = 0.169676\ldots$ of the global coherence; agreement with Planck 2018 [10] $\Omega_\Lambda = 0.6889 \pm 0.0056$ within 0.05σ *without fitting*.

These two anchor formulas — XL.F2 (structural keystone) and XL.F3 (numerical keystone) — are the main synthetic results of the present work. They are not re-derived in XL: XL.F2 is a reformulation of C.T1 from [16] §VI, XL.F3 is a reformulation of B.F23 from [15] §VIII. Their joint demonstration in one paper completes the programme notation.

VI. SCHWARZSCHILD AS AN EXACT ODTOE SOLUTION (SYNTHESIS OF A.T4 + C.T1 VACUUM LIMIT)

VI.1. Schwarzschild as a fixed point of Φ_C

The Schwarzschild metric (formula F11 from [14]):

$$ds^2_{\text{Schw}} = -\left(1 - \frac{r_s}{r}\right) c^2 dt^2 + \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 + r^2 d\Omega^2, \qquad r_s = \frac{2GM}{c^2} \tag{6.1}$$

is a fixed point of Φ_C in C_{contr} at $T = 0$, $\Lambda = 0$ (Theorem A.T4 from [14] §VIII.1 + statement from [16] §VIII.1). Proof: by A.T4 for (6.1) one has $R_{\mu\nu} = 0$ in vacuum, hence $G_{\mu\nu} = 0$ identically; application of $\hat{O}$ from formula (6.1) of [16] to g_{Schw} gives $T_{\mu\nu} = 0$; uniqueness of the Schwarzschild solution with given $T = 0$ (Birkhoff theorem [18] §6.1) gives $\iota(T = 0) = g_{\text{Schw}}$ modulo Diff. Composition $\Phi_C(g_{\text{Schw}}, 0) = (g_{\text{Schw}}, 0)$.

VI.2. Numerical verification of Schwarzschild

Numerical verification (perihelion shift of Mercury test from [14] §IX.1):

$$\Delta\phi_{\text{century}} = 42.9916585896956795 \text{ arcsec/century} \tag{6.2}$$

in full agreement with the experimental value 42.98 ± 0.04 arcsec/century [19]. This is the first verification of theorem T0 (see §IX) on a concrete solution.

VII. KERR VERIFIED (CITATION OF A.T5)

VII.1. Kerr as a fixed point of Φ_C

The Kerr metric in Boyer–Lindquist coordinates [8]:

$$\begin{aligned} ds^2_{\text{Kerr}} = &-\left(1 - \frac{r_s r}{\Sigma}\right) c^2 dt^2 - \frac{2 r_s r a c \sin^2\theta}{\Sigma} dt\, d\phi + \frac{\Sigma}{\Delta} dr^2 + \Sigma\, d\theta^2 \\ &+ \left(r^2 + a^2 + \frac{r_s r a^2 \sin^2\theta}{\Sigma}\right) \sin^2\theta\, d\phi^2 \end{aligned} \tag{7.1}$$

with $\Sigma = r^2 + a^2\cos^2\theta$, $\Delta = r^2 - r_s r + a^2$, $a = J/(Mc)$ — rotation parameter, J — angular momentum. The outer horizon and equatorial ergosphere are given by the explicit expressions [14] equations (8.2)–(8.3):

$$r_+ = M + \sqrt{M^2 - a^2}, \qquad r_E^{\text{eq}} = 2M = r_s \tag{7.2}$$

By Theorem A.T5 from [14] §VIII.2 the pair $(g_{\text{Kerr}}, T = 0)$ satisfies $R_{\mu\nu} = 0$ in vacuum (standard result of Kerr theory [7, 8]), hence $\Phi_C(g_{\text{Kerr}}, 0) = (g_{\text{Kerr}}, 0)$ — Kerr is a fixed point of Φ_C for a rotating source [16] §IX.

VII.2. Vortex SYNC component and ODTOE interpretation

In the ODTOE interpretation, the off-diagonal component $g_{t\phi} = -r_s rac\sin^2\theta/\Sigma$ corresponds to a vortex SYNC component induced by the angular momentum of the source: the rotation of the massive body generates a local twisting of SYNC actualization fronts, which in the macro-limit recovers the classical frame-dragging effect [7] §33. Numerical verification of r_+ and r_E^{eq} at 50-digit precision is given in [14] §IX.2 (formulae (9.6)–(9.8)) and not repeated here.

VIII. FLRW AND COSMOLOGICAL CLOSURE (USE OF $\chi_\Lambda(S^*)$ FROM B)

VIII.1. Friedmann equation from Φ_C-fixedness

For the spatially homogeneous isotropic FLRW metric

$$ds^2_{\text{FLRW}} = -c^2dt^2 + a(t)^2\left[\frac{dr^2}{1-kr^2} + r^2d\Omega^2\right] \tag{8.1}$$

with scale factor $a(t)$ and curvature $k \in \{-1, 0, +1\}$, the Φ_C-fixedness of the pair $(g_{\text{FLRW}}, T_{\text{cosm}})$ gives the Friedmann equation (formula C.F17 from [16]):

$$H^2 = \frac{8\pi G}{3}\rho_{\text{tot}} - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}, \qquad H = \dot{a}/a \tag{8.2}$$

with $\rho_{\text{tot}} = \rho_m + \rho_r + \rho_\Lambda$.

VIII.2. Substitution of $\chi_\Lambda(S^*)$ and comparison with Planck 2018

From the closed form of $\chi_\Lambda(S^*)$ (formula B.F23) and the identity $\Omega_\Lambda = \varphi^2/(\varphi^2 + 1 + Z(S^*))$ with substitution of 50-digit constants (see [15] §VIII.4 steps 1–3):

$$\begin{aligned}
\pi &= 3.14159265358979323846264338327950288419716939937510\\
\varphi &= 1.61803398874989484820458683436563811772030917980576\\
(\pi - 3) &= 0.14159265358979323846264338327950288419716939937510\\
\varphi^2 &= 2.61803398874989484820458683436563811772030917980576\\
Z(S^*) &= 0.18367229293062031020\ldots\\
\Omega_\Lambda(S^*) &= 0.68864709548066742428\ldots
\end{aligned}$$

Comparison with Planck 2018 [10] $\Omega_\Lambda = 0.6889 \pm 0.0056$:

$$|\Omega_\Lambda^{\text{Planck}} - \Omega_\Lambda(S^*)| = 0.00025290\ldots < 0.0056 = 1\sigma \quad \Rightarrow \quad 0.05\sigma \text{ deviation} \tag{8.3}$$

without fitting — this is the second numerical keystone of programme T0.

VIII.3. Honest reservation: vacuum-trivial Path 2 on FLRW

Numerical verification of the dual-path Bianchi identity (Theorem C.T2) was performed in [16] §V.4 on the Schwarzschild ground state, where $T_{\mu\nu} = 0$ ensures the trivial agreement of Path 1 and Path 2 (both give $\nabla_\mu G^{\mu\nu} = 0$ automatically). The tensor proof of Bianchi in C.T2 is structurally complete (proved through Noether symmetry and the kinematic identity A.T3, see [16] §V.3); however, *the numerical verification of Path 2 on a non-trivial FLRW background with $T_{\mu\nu} \neq 0$ is left as an open task* (see [16] §XI item ii). The present paper documents the completion of the programme on the basis of the full structural derivation; the non-trivial numerical verification is a direction for future publication, in which Path 2 will be checked on FLRW with realistic densities of matter ρ_m, radiation ρ_r, and dark energy ρ_Λ. This item *does not block* the closure of the programme by T0, since the structural proof of C.T2 does not depend on the choice of background solution.

IX. PROGRAMME COMPLETION THEOREM T0

IX.1. Statement of T0

Theorem T0 (Programme Completion). *The combined results of articles [A] = [14], [B] = [15], and [C] = [16] suffice to derive the full dynamical Einstein equation*

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{T0}$$

from ODTOE primitives in the following sense:

1. **[A] = [14]** *supplies $g_{\mu\nu}$ as observer-correlator (formula A.F1), ∇_μ as the limit of the Φ-iteration commutator (formula A.F3), $R^\rho{}_{\sigma\mu\nu}$ via non-commutativity of SYNC operations (formula A.F5), $R_{\mu\nu}$, R, $G_{\mu\nu}$ explicitly through standard contractions (formulae A.F7–A.F9), with notation Π_I for the inertial scalar potential and the Kerr solution derived as a spherically-axial SYNC-vortex ansatz. Theorems A.T1–A.T5 are PROVED in [14].*

2. **[B] = [15]** *supplies $T_{\mu\nu} = \delta S_{\text{obs}}/\delta g^{\mu\nu}$ via the SYNC projector $P_{O,\text{SYNC}}$ with PROVED idempotency (lemma L7) and PROVED conservation law (lemma L8), and Λ via the closed form $\chi_\Lambda(S^*) \approx 0.082201$, giving $\Omega_\Lambda \approx 0.688647$ in agreement with Planck 2018 [10] within 0.05σ.*

3. **[C] = [16]** *supplies Theorem C.T1 on Φ-self-consistency (PROVED), Theorem C.T2 on the dual-path Bianchi identity $\nabla_\mu G^{\mu\nu} = 0$ as Noether consequence of diffeomorphism invariance (PROVED), and ODTOE analog of the Hawking–Penrose singularity theorem C.T3 (PROVED with status "proved with explicitly indicated open task on the topology of the boundary point $B \to 0$," see [16] §VII.5).*

4. **Standard solutions (Schwarzschild, Kerr, FLRW)** *are recovered as exact ODTOE constructions, not as ansätze; see §VI–§VIII of the present work and [14] §VIII–§IX, [15] §VIII, [16] §VIII–§X.*

5. **Programme §XIV.3, declared open in [13],** *is thereby semantically closed; the original disclaimer formulation in [13] §I (lines 117–120) is a historical artifact reflecting the state prior to completion of the present synthetic work (see detailed discussion in §X).*

*Proof. T0 is a **synthetic statement**, not a new theorem. The proof is the chain itself: §A→ §B→ §C→ §XL. Formally, each of the statements (i)–(v) is a reference to the corresponding theorem/lemma of an already published result:*

- *Statement (i) — theorems A.T1 (Levi-Civita connection), A.T2 (Riemann properties), A.T3 (kinematic Bianchi identity), A.T4 (Schwarzschild), A.T5 (Kerr) are proved in [14].*
- *Statement (ii) — lemma L7 (idempotency) is proved in [15] §V; lemma L8 (conservation) is proved in [15] §VII; closed form $\chi_\Lambda(S^*)$ is derived in [15] §VIII.*
- *Statement (iii) — Theorem C.T1 (Φ-self-consistency) is proved in [16] §VI; Theorem C.T2 (dual-path Bianchi identity) is proved in [16] §IV–§V; Theorem C.T3 (ODTOE singularities) is proved in [16] §VII with explicit reservation §VII.5.*
- *Statement (iv) — Schwarzschild as a fixed point of Φ_C is proved in [16] §VIII.1, Kerr in §IX, FLRW in §X.*
- *Statement (v) — programme observation, grounded in §X of the present work.*

The combination gives the full chain of derivation of (1.1) from ODTOE primitives. Statements (i)–(v) require no independent proofs — all of them are already proved in sources [14], [15], [16]; XL combines them into a formal unit. □

IX.2. Honest reservation: open tasks within T0

The tensor proof of Bianchi in C.T2 is structurally complete; the numerical verification of Path 2 on a non-trivial FLRW background with $T_{\mu\nu} \neq 0$ is left as an open task (see [16] §XI item ii). The present paper documents the completion of the programme on the basis of the full structural derivation; the non-trivial numerical verification is a direction for future article. This reservation does not undermine T0, since: (a) the structural proof of C.T2 rests on Noether symmetry and Lovelock's theorem [5] on the uniqueness of $G_{\mu\nu}$; (b) the vacuum numerical verification on Schwarzschild (formula C.F9) in 50-digit arithmetic gives exact agreement of the two paths; (c) extension to non-trivial backgrounds is a technical strengthening, not a structural gap.

IX.3. What T0 closes and what remains open

Closed by T0:

- Derivation of $g_{\mu\nu}$ from the self-observation operator Φ — Theorem A (see formula A.F1).
- Derivation of ∇_μ from the Φ-iteration commutator — Theorem A.T1 (see formula A.F3).
- Derivation of $R^\rho{}_{\sigma\mu\nu}$, $R_{\mu\nu}$, R, $G_{\mu\nu}$ from the non-commutativity of SYNC — Theorems A.T2–A.T3.
- Kinematic Bianchi identity — Theorem A.T3 (Path 1 for C.T2).
- Derivation of $T_{\mu\nu}$ from the B-functional — formula B.F15.
- Idempotency of the SYNC projector — lemma L7 PROVED.
- Conservation law $\nabla_\mu T^{\mu\nu} = 0$ — lemma L8 PROVED.
- Closed form of the cosmological constant $\chi_\Lambda(S^*)$ — formula B.F23.
- Equivalence of the Einstein equation to Φ-self-consistency — Theorem C.T1 PROVED.
- Dynamical Bianchi identity as Noether consequence — Theorem C.T2 Path 2 PROVED.
- ODTOE analog of the Hawking–Penrose theorem — Theorem C.T3 PROVED with honest [OPEN: $B \to 0$ boundary topology].
- Schwarzschild, Kerr, FLRW as exact ODTOE fixed points of Φ_C.
- Agreement of Ω_Λ with Planck 2018 within 0.05σ *without fitting*.

Remains open (for future articles):

- Full topological formalization of the limit $B \to 0$ as a boundary point of the Φ-iteration (see [16] §XI item i).

- Analytical numerical verification of Path 2 on a non-trivial FLRW with $T_{\mu\nu} \neq 0$ (see [16] §XI item ii).
- ODTOE formulation of smoothness and causality conditions for Φ-iteration sequences near horizons and singularities (see [16] §XI item iii).
- Integration with the thermodynamic derivation of [15] §IX through horizon ODTOE analogs of Hawking–Ellis [9] theorems and of Jacobson's [11] thermodynamic Einstein equation of state (horizon as $\delta Q = T\,dS$, which in ODTOE is reformulated as a Fix(Φ) condition on J_O^+); see [16] §XI item iv.

These open tasks define the forward programme of ODTOE gravity beyond the initial four-article programme.

X. RELATION TO [13]

X.1. Historical role of work [13]

The work [13] *ODTOE_gravity_causal_structure* occupies a special position in programme §XIV.3: it is the *first* article formalizing the causal layer of ODTOE gravity as stage 1 of the derivation. Its §VI introduced the relation of causal reachability of configurations $C_i \preceq_O C_j$, the causal cone J_O^+, and the effective metric $g_{00}^{\text{eff}} = (I_0/I_{\text{eff}})^2$, on which all subsequent works of the programme rest: § A [14] extends g_{00}^{eff} to the full tensor $g_{\mu\nu}$ via the observer-correlator; § B [15] uses the causal layer $\mathcal{C} \subset \mathcal{H}$ as the image of the SYNC projector; § C [16] rests on J_O^+ for the definition of the contraction subspace C_{contr} in Theorem C.T1.

X.2. Disclaimer §I as historical artifact

The present work documents the completion of programme §XIV.3, declared in [13]. The disclaimer in [13] §I (lines 117–120) is semantically retired by this synthesis — the programme is performed. However, the work [13] itself remains the canonical formalization of the causal layer as stage 1 of the derivation; its disclaimer formulation and §XIV.3 "Open programme" historically fix the state prior to completion of the programme. A reader consulting [13] should interpret these formulations in the context of the completed programme documented in the present work.

X.3. Work [13] is not modified

Within the framework of the present XL work, *no modifications* are made to source [13]: the disclaimer §I and §XIV.3 remain in their original formulation. This choice is made *deliberately* — to preserve the status integrity of the programme cycle and the atomicity of the commit of the present XL paper. The citation chain ensures correct interpretation: a future reader, opening [13], follows the reference [16] (which, in turn, refers to the present XL) and obtains the full description of the programme state.

X.4. Citation chain for completion status

Chain for the future reader:

- Opening [13], the reader sees the disclaimer §I and the open-programme statement of §XIV.3.
- The reference chain §XIV.3 points to stage 1 (causal layer, performed in [13]); the subsequent stages 2–3 are formally open in the formulation of [13].
- The work [14] closes stage 1 in the full tensor sense and explicitly indicates stages 2–3 as next steps.
- The work [15] closes stage 2 and indicates stage 3.
- The work [16] closes stage 3 and formulates the three-stage programme as closed (see [16] §XI conclusion).
- The present XL work formulates Theorem T0 (see §IX) as the final closure of the programme and explicitly describes the status of the disclaimer [13] §I (see §X.2).

Thus, programme §XIV.3 is completed: the chain § A→ § B→ § C→ § XL fixes all three stages. The disclaimer [13] §I, while not modified, is correctly interpreted in the context of completion through the reference to the present XL work.

XI. POST-EINSTEIN OUTLOOK AND FUTURE PROGRAMMES

XI.1. Quantum gravity in ODTOE

The completion of programme §XIV.3 closes the classical layer of ODTOE gravity. *The next level* — quantum gravity in ODTOE — requires extending the Φ-iteration structure to the Hilbert quantization of the self-observation operator $\hat{O}$. Natural directions: (i) ODTOE analog of loop quantum gravity [20] via discretization of SYNC fronts on scales r_0, τ_0 from [13] equation (2.6); (ii) theory of Φ-iteration path integral as ODTOE analog of the Feynman formalism for gravity; (iii) extension of the SYNC projector $P_{O,\text{SYNC}}$ to a quantum channel with Kraus operator elements.

XI.2. ODTOE-string and string geometry

Structural conjecture: SYNC fronts of actualization on the φ-torus from [12] §VIII can be reformulated as one-dimensional extended objects (strings) in the Hilbert layer $\mathcal{H}$. Potential connection with string theory — through identification $r_0 = l_s$ (characteristic string length) [21]. This conjecture requires independent mathematical elaboration and is explicitly assigned to the forward programme.

XI.3. Consciousness–gravity link

The third direction is the link of consciousness and gravity through the ODTOE coherence parameter $B(O,C)$. The work [22] *ODTOE_dynamic_attractor* derives a dynamic attractor as a structural model of evolutionary monadology; in the present XL work this direction is mentioned only as a HYPOTHESIS. Possible tests: (i) correlation of the global coherence S^* with cosmological parameters Hubble tension and S_8 [23]; (ii) connection of the parameter B with observer entropy through the thermodynamic horizon derivation [15] §IX. This direction requires significant experimental verification before transition to derivation status.

XI.4. Post-Einstein extensions

The closure of programme §XIV.3 does not preclude post-Einstein extensions of equation (1.1). Possible directions: (i) ODTOE analog of $f(R)$-gravity through nonlinear action $S^{(n)}_{\text{obs}} = \int F[B^2(1-\sigma)\Lambda]\sqrt{-g}\,d^4x$ for nonlinear F; (ii) tensor-scalar modifications through inclusion of Π_I as a dynamical variable in the action; (iii) Lovelock extensions [5] of higher derivatives through ODTOE formulation. Each of these directions is a self-contained task of a separate publication.

XI.5. Forward programme as a summary list

Forward programme of ODTOE gravity (after completion of §XIV.3):

1. Topology of the limit $B \to 0$ for C.T3 (from [16] §XI item i).
2. Numerical verification of Path 2 on non-trivial FLRW (from [16] §XI item ii).
3. Smoothness and causality conditions near horizons (from [16] §XI item iii).
4. Integration with horizon thermodynamics [9] (from [16] §XI item iv).
5. Quantum gravity in ODTOE (new direction; see §XI.1 of the present work).
6. ODTOE-string conjecture (new; see §XI.2).
7. Consciousness–gravity link (new speculative direction; see §XI.3).
8. Post-Einstein extensions (new; see §XI.4).

XII. CONCLUSION

In the present work, Theorem T0 on the completion of programme §XIV.3 of [13] is formulated and grounded for the full derivation of the Einstein equation $G_{\mu\nu} + \Lambda g_{\mu\nu} = (8\pi G/c^4)T_{\mu\nu}$ from ODTOE primitives. The programme is realized by a four-article cycle:

- **[13] = ODTOE_gravity_causal_structure** — stage 1, causal layer; formulation of programme §XIV.3.
- **[14] = ODTOE_gravity_tensor_structure (Article A)** — tensor structure: $g_{\mu\nu}$, ∇_μ, $R^\rho{}_{\sigma\mu\nu}$, $G_{\mu\nu}$, theorems A.T1–A.T5.
- **[15] = ODTOE_gravity_T_munu_projector (Article B)** — tensor source: $T_{\mu\nu}$, $P_{O,\mathrm{SYNC}}$, L7, L8, $\chi_\Lambda(S^*)$.
- **[16] = ODTOE_einstein_derivation_complete (Article C)** — closure: C.T1, C.T2, C.T3.
- **The present XL paper** — synthesis T0 and formal fixation of programme closure.

The main methodological result is the *synthetic nature* of the ODTOE derivation of the Einstein equation. Programme §XIV.3, declared in [13] §I as open, is performed in full: each of the three structural stages is realized by a separate paper with explicit anti-circularity audit and numerical verification in 50-digit arithmetic. Standard solutions (Schwarzschild, Kerr, FLRW) are recovered as exact ODTOE fixed points of Φ_C, not as ansätze.

The disclaimer [13] §I (lines 117–120) is preserved in its original formulation as a historical artifact; the citation chain § A→ § B→ § C→ § XL ensures correct interpretation of the completed programme for the future reader. The forward programme of ODTOE gravity — topology of $B \to 0$, non-trivial FLRW Path 2, smoothness conditions near horizons, horizon thermodynamics, quantum gravity, ODTOE-string, consciousness–gravity link, post-Einstein extensions — defines the directions for further publications of the corpus.

The programme A→B→C→XL is closed. The Einstein equation is derived from ODTOE primitives. Theorem T0 is PROVED as a synthetic statement, the proof of which is the derivation chain itself.

ACKNOWLEDGEMENTS AND TOOLS

The author thanks the community of researchers of observer-dependent interpretations of general relativity and quantum mechanics for discussions of key ideas of the derivation of the Einstein equation as Φ-self-consistency and reviews of the structural synthesis of the four-article programme. The numerical verifications §VI.2, §VIII.2 rest on calculations performed within articles [14], [15], [16] using the `mpmath` library (arbitrary precision for Python; `mp.dps=60` for 50-digit arithmetic). Text preparation was carried out using the LaTeX distribution `tectonic` (XeLaTeX-compatible compiler), `pandoc` for generation of `.docx` and `.md` formats, and AI editing tools. All scientific responsibility for the content rests with the author.

CONFLICT OF INTEREST

The author declares no conflict of interest in relation to the content of the present work.

FUNDING

The present research did not receive external funding. The work was carried out as an independent research initiative.

REFERENCES

Note on order. The references are organized in three conceptual blocks [L-35-ext]: (1) foundational classical works (Reed–Simon, Noether, Penrose, Hawking–Penrose, Lovelock, Banach, MTW, Boyer–Lindquist, Hawking–Ellis, Planck, Jacobson, Wald, Will, Rovelli, Polchinski, Riess) — by conceptual order; (2) author's preprints in the ODTOE corpus — by first citation in the text. The reference data block is absent, since the present article is synthetic (Theorem T0 as a structural statement).

1. Reed, M., Simon, B. *Methods of Modern Mathematical Physics, vol. I: Functional Analysis*. Academic Press (1980). ISBN: 0-12-585050-6. (Theorem II.3 on orthogonal projection.)

2. Noether, E. Invariante Variationsprobleme. *Nachr. v.d. Ges. d. Wiss. zu Göttingen, math.-phys. Klasse*, 235–257 (1918). EN translation: Tavel, M.A. Invariant variation problems. *Transport Theory and Statistical Physics* 1, 186–207 (1971). DOI: 10.1080/00411457108231446.

3. Penrose, R. Gravitational collapse and space-time singularities. *Phys. Rev. Lett.* 14(3), 57–59 (1965). DOI: 10.1103/PhysRevLett.14.57.

4. Hawking, S.W., Penrose, R. The singularities of gravitational collapse and cosmology. *Proc. Roy. Soc. Lond. A* 314, 529–548 (1970). DOI: 10.1098/rspa.1970.0021.

5. Lovelock, D. The Einstein tensor and its generalizations. *J. Math. Phys.* 12(3), 498–501 (1971). DOI: 10.1063/1.1665613.

6. Banach, S. Sur les opérations dans les ensembles abstraits et leur application aux équations intégrales. *Fundamenta Mathematicae* 3, 133–181 (1922). DOI: 10.4064/fm-3-1-133-181.

7. Misner, C.W., Thorne, K.S., Wheeler, J.A. *Gravitation*. W.H. Freeman (1973). ISBN: 0-7167-0344-0. (Princeton reprint 2017, ISBN: 978-0-691-17779-3.) (MTW sign convention.)

8. Boyer, R.H., Lindquist, R.W. Maximal analytic extension of the Kerr metric. *J. Math. Phys.* 8(2), 265–281 (1967). DOI: 10.1063/1.1705193.

9. Hawking, S.W., Ellis, G.F.R. *The Large Scale Structure of Space-Time*. Cambridge University Press (1973). ISBN: 0-521-09906-4.

10. Planck Collaboration: Aghanim, N. et al. Planck 2018 results. VI. Cosmological parameters. *Astron. Astrophys.* 641, A6 (2020). DOI: 10.1051/0004-6361/201833910. (Used $\Omega_\Lambda = 0.6889 \pm 0.0056$.)

11. Jacobson, T. Thermodynamics of spacetime: The Einstein equation of state. *Phys. Rev. Lett.* 75(7), 1260–1263 (1995). DOI: 10.1103/PhysRevLett.75.1260.

12. Wald, R.M. *General Relativity*. The University of Chicago Press (1984). ISBN: 0-226-87033-2. (Standard GR textbook.)

13. Pankratov, A.S. *Gravity and the Causal Structure of Spacetime in ODTOE*. Preprint (2026). Slug: `ODTOE_gravity_causal_structure`.

14. Pankratov, A.S. *Tensor Structure of Gravity in ODTOE*. Preprint (2026). Slug: `ODTOE_gravity_tensor_structure`.

15. Pankratov, A.S. *Stress-Energy Tensor $T_{\mu\nu}$ and Cosmological Constant Λ from Observer Coherence in ODTOE*. Preprint (2026). Slug: `ODTOE_gravity_T_munu_projector`.

16. Pankratov, A.S. *Einstein Equation as Φ-Self-Consistency and Bianchi Identity from Diff(M^4) Symmetry in ODTOE*. Preprint (2026). Slug: `ODTOE_einstein_derivation_complete`.

17. Pankratov, A.S. *Infinite Recursion and the Unified Self-Observation Operator*. Preprint (2026). Slug: `ODTOE_infinite_recursion_unified`.

18. Pankratov, A.S. *Observer-Dependent Theory of Everything*. Preprint (2026). Slug: `ODTOE_article`.

19. Will, C.M. The confrontation between general relativity and experiment. *Living Rev. Relativity* 17, 4 (2014). DOI: 10.12942/lrr-2014-4. (Modern GR tests.)

20. Rovelli, C. *Quantum Gravity*. Cambridge University Press (2004). ISBN: 0-521-83733-2. (Loop quantum gravity.)

21. Polchinski, J. *String Theory, vol. I*. Cambridge University Press (1998). ISBN: 0-521-63303-6. (Modern presentation of string theory.)

22. Pankratov, A.S. *Dynamic Attractor in ODTOE: Evolutionary Monadology and Energy-Information Density of the World Line*. Preprint (2026). Slug: `ODTOE_dynamic_attractor`.

23. Riess, A.G. et al. A 2.4% determination of the local value of the Hubble constant. *Astrophys. J.* 826, 56 (2016). DOI: 10.3847/0004-637X/826/1/56. (Hubble tension context for XI.3.)

24. Pankratov, A.S. *The Unified Self-Observation Operator: From Physical Constants Through Toroidal Geometry to Language Structure*. Preprint (2026). Slug: `ODTOE_unified_operator`.

25. Pankratov, A.S. *Earth as a Cluster of Observers: Universe Synchronization in ODTOE*. Preprint (2026). Slug: `ODTOE_collective_observer`.

26. Pankratov, A.S. *Gravity as Observer Synchronization: Deriving the Gravitational Constant from ODTOE First Principles under the Structural Hypothesis* $C = B^2$. Preprint (2026). Slug: `ODTOE_gravity_v2`.

27. Pankratov, A.S. *Planck's Constant from ODTOE: Deriving* $\hbar$ *from Toroidal Geometry and Observer Coherence*. Preprint (2026). Slug: `ODTOE_planck_constant`.