A GEOMETRIC RESOLUTION OF THE HUBBLE TENSION:

DARK-ENERGY AND DARK-MATTER UNIFICATION VIA PARENT-PROTON MERGERS IN THE ODTOE MATRYOSHKA

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ABSTRACT

We present a geometric mechanism that simultaneously resolves the cosmological constant problem and the H_0 tension within a single one-parameter framework. The **Postulate of Geometric Primacy (GP)** (Section III.0, here introduced as Postulate P7) fixes the asymptotic dark-sector attractor $\varphi^2 : 1 : Z$ as a topological invariant; dynamics modulates only the rate of approach. The Hubble tension between the Planck 2018 inferred value $H_0 = 67.4 \pm 0.5$ km s^{-1} Mpc^{-1} [1] and the SH0ES local determination $H_0 = 73.04 \pm 1.04$ km s^{-1} Mpc^{-1} [2] currently stands at the $\sim 5\sigma$ level (range 4.0–5.8σ across late-Universe anchor combinations [9]) and remains unexplained within standard ΛCDM [3]. The Observer-Dependent Theory of Everything (ODTOE) treats the visible Universe as one level $d = 9$ in a recursive matryoshka of nested φ-tori [21], whose static cosmological fractions $\Omega_\Lambda : \Omega_{DM} : \Omega_b = \varphi^2 : 1 : Z = 68.86\% : 26.30\% : 4.83\%$ already match Planck within 1–2σ [21]. The present paper extends that static derivation to a dynamical mechanism. Dark energy is identified with the merger process by which parent-protons — coherent structures at level $d = 12$ for which our universe is a single proton at level $d = 9$ — gradually combine through the geometric channel of the 2%-spiral residue $(\pi - 3)^2 \approx 0.0200485$. The merger rate is regulated by a scalar field $\chi(x, t)$ with background value χ_0 and small spatial fluctuations $\Delta\chi$. Three independent claims follow: (a) χ-regimes (slow, medium, fast) classify expansion histories without modifying the geometric attractor; (b) anisotropic $\Delta\chi$ between early-Universe (CMB) and local patches reproduces the observed H_0 tension; (c) the dark sector itself is unified, with Ω_{DM} and Ω_Λ being two aspects of the same 2%-residue accumulated over N merger steps. Five falsifiable predictions are stated. A merger limit $N_{\max}^{(\mathrm{local})} = \Omega_{DM}/(\pi - 3)^2 \approx 13.12$ is derived as the saturation of the local octave, beyond which the system performs an octave shift $d \to d + 9$ in the matryoshka hierarchy — bounded local mergers reconciled with eternal global expansion. Postulate P7 (GP) ensures that any χ-history asymptotes to the static attractor $(\varphi^2 : 1 : Z)/\Sigma$, eliminating double counting between geometry and dynamics. The model contains a single fitting parameter η in the merger-rate prefactor; all other quantities follow from the topological invariants φ, π, Z. **Main contribution:** reformulation of dark energy from a *composition* parameter into a *process* with fixed geometric asymptotics $\varphi^2 : 1 : Z$ and a single fitting parameter η.

Keywords: dark energy, **Postulate of Geometric Primacy (GP)**, dark matter, Hubble tension, parent-proton merger, ODTOE, χ-field, matryoshka recursion, φ-torus, 2%-spiral residue, H_0 anisotropy, octave shift, merger limit, falsifiable predictions.

I. INTRODUCTION

I.1. The Hubble tension and the missing-mechanism critique

The Hubble tension between Planck [1] and SH0ES [2] has reached the 4.0–5.8σ range [9] and remains unexplained within ΛCDM. In addition, the cosmological constant Λ in the standard ΛCDM model is treated as a static parameter fitted to observations, with no derivation from first principles: the discrepancy of $\sim 10^{120}$ between the quantum-field-theoretic vacuum-energy estimate and the observed dark-energy density is the unresolved cosmological constant problem [3, 4]; the contemporary dark-sector landscape is reviewed in detail in [17]. The two problems — the tension problem and the magnitude problem — are addressed jointly below.

The Planck 2018 analysis of the cosmic microwave background (TT,TE,EE+lowE+lensing) yields $H_0 = 67.4 \pm 0.5$ km s^{-1} Mpc^{-1} and $\Omega_\Lambda = 0.6889 \pm 0.0056$ (Table 2, TT,TE,EE+lowE+lensing column) [1]. The SH0ES Cepheid–SN distance ladder gives $H_0 = 73.04 \pm 1.04$ km s^{-1} Mpc^{-1} [2]. Combining the late-Universe distance-ladder anchors with Planck CMB-inferred values yields a tension between 4.0σ and 5.8σ depending on which three independent late-Universe approaches are combined [9]; the comprehensive solution survey of Di Valentino et al. [10] catalogs the candidate explanations, none of which has gained universal acceptance.

Standard ΛCDM with a single static Λ has no internal mechanism that distinguishes early- from late-Universe expansion rates. Phantom dark energy with $w < -1$ [5] resolves part of the tension, but at the cost of a finite-time Big Rip, which contradicts the bounded asymptotic fractions found in toroidal ODTOE [21]. We argue below that a dynamical merger mechanism, regulated by a scalar χ-field but constrained by a fixed geometric attractor, resolves both the magnitude and tension problems without requiring $w < -1$ and without invoking modified gravity.

I.2. ODTOE matryoshka and prior cosmological derivation

The ODTOE framework [24] models reality as a hierarchy of nested φ-tori. Each level d is a torus with $R/r = \varphi$, maximally stable by the KAM theorem [18]. The toroidal cosmology paper [21] derives the present-day cosmological fractions purely from π and φ:

$$\Omega_\Lambda : \Omega_{DM} : \Omega_b = \varphi^2 : 1 : Z, \qquad Z = \frac{\pi - 3}{1 - (\pi - 3)\varphi}, \tag{I.1}$$

giving $\Omega_\Lambda = 68.86\%$, $\Omega_{DM} = 26.30\%$, $\Omega_b = 4.83\%$, in agreement with Planck [1] within 1–2σ (the standard modern-cosmology baseline against which these fractions are compared is laid out in [13]). This derivation has zero adjustable parameters but is purely static — it tells us what fractions *are*, not how they evolve.

I.3. The merger hypothesis and structure of the paper

We propose that dark energy is the macroscopic signature of a merger process: at level $d = 12$, parent-protons are coherent structures for which our universe at $d = 9$ is itself a single proton; these parent-protons gradually merge through the geometric channel of the 2%-spiral residue $(\pi - 3)^2 \approx 0.0200485$. The rate is controlled by a scalar field $\chi(x, t)$. The 2%-residue plays the same role here as in the static derivation of Ω_b in [21]: it is the geometric remainder of the unclosed observation loop $\pi > 3$, accumulated over an evolving number of merger acts N.

The paper proceeds as follows. Section II reviews the ODTOE matryoshka basis. Section III.0 introduces the postulate of geometric primacy. Section III defines the merger kinetics and derives the cumulative residue formula. Section IV catalogs the topological invariants φ^2, $(\pi - 3)^2$, Z. Section IV.5 verifies that the static formula (I.1) is recovered as a fixed point. Sections V–VII develop the three independent claims: χ-regimes, H_0 anisotropy, and DE–DM unification. Section VIII states five falsifiable predictions. Section VIII.5 derives the merger limit $N_{\text{max}}^{\text{(local)}} \approx 13.12$ and the octave-shift mechanism. Section IX is the demarcation table; Section X discusses open issues.

II. ODTOE MATRYOSHKA BASIS

II.1. Recursive nesting and level d

The matryoshka hierarchy assigns to each level d a φ-torus of major-to-minor radius ratio $R/r = \varphi$. The atom corresponds to $d = 0$; our observable universe corresponds to $d = 9$. The base of nine octave steps from $d = 0$ to $d = 9$ is fixed by the discrete iterative dynamics of the self-observation loop [24, 28]. The standard cosmological background needed to interpret these levels (FRW expansion, perturbation theory, decoupling sequence) is provided by the canonical textbook treatment [11]. A parent-proton at $d = 12$ is then a coherent structure three octave steps above our universe, for which our universe is itself a single proton-scale constituent.

II.2. The φ-torus and KAM stability

The trajectory on the φ-torus is described by two angular coordinates θ (rotation around the minor radius) and ϕ (rotation around the major radius). When $\omega_\theta/\omega_\phi = R/r = \varphi$ (the most irrational number [19]), the KAM theorem [18] guarantees maximal stability of the quasi-periodic motion against small perturbations. The trajectory is dense on the surface and never closes; this non-closure is the source of the spiral gap.

II.3. Cosmological fractions and the 2%-spiral gap

The static cosmological fractions of [21] are:

$$\Omega_\Lambda = \frac{\varphi^2}{\Sigma} = 68.86\%, \quad \Omega_{DM} = \frac{1}{\Sigma} = 26.30\%, \quad \Omega_b = \frac{Z}{\Sigma} = 4.83\%, \tag{II.1}$$

with $\Sigma = \varphi^2 + 1 + Z$.

The first-order spiral gap is $\delta_1 = \pi - 3$. The second-order residue, which we will use as the merger channel, is

$$\varepsilon \equiv (\pi - 3)^2 = 0.020048479550599188058630700199 13\ldots \tag{II.2}$$

This is the gap of the gap: the residual mismatch after one full attempt to close the spiral [21, 28]. Its independent status as a postulate (rather than a derived quantity) is asserted explicitly below.

III.0. POSTULATE P7 (GEOMETRIC PRIMACY)

Postulate P7 (Geometric Primacy, GP). For any $\chi(x,t)$-history compatible with positive merger rates and bounded total mass, the cosmological fractions satisfy

$$\lim_{t\to\infty} \Omega_i(t \mid \chi) \;=\; \Omega_i^{(\text{geom})} \;=\; \left\{\frac{\varphi^2}{\Sigma},\, \frac{1}{\Sigma},\, \frac{Z}{\Sigma}\right\}, \tag{III.0.1}$$

independently of the path of χ.

Geometry sets the asymptotic state; dynamics modulates only the rate of approach. The corollary is that the three claims developed below — χ-regimes (Section V), $\Delta\chi$-anisotropy (Section VI), and DE–DM unification via the 2%-residue (Section VII) — are mutually *independent* and do not double-count: χ-regimes change *when* the system reaches the attractor; $\Delta\chi$ is the local fluctuation around the attractor at finite redshift; the 2%-residue is the geometric channel through which the rate operator acts. Without GP, claims a, b, c could in principle interact non-trivially; GP guarantees a clean factorization.

Parity test. The derivation of Section IV.5 verifies, to the 50-digit precision of φ^2, Z, and Σ, that as $\chi \to \chi_0$ and $\Delta\chi \to 0$ the present model reproduces equation (II.1) exactly — the numerical residual $|\Omega_\Lambda^{(\text{this})} - \Omega_\Lambda^{(21)}| < 10^{-40}$.

Operational falsifier of Postulate P7. Postulate P7 is refuted if any of the following observational signatures is established at 5σ:

(a) confirmed late-time excursion $w(z) < -1$ at $\geq 3\sigma$ in any DESI Y5 redshift bin;

(b) measured drift $|\Delta(\Omega_\Lambda + \Omega_{DM} + \Omega_b) - 1| > 0.01$ at $z < 0.5$;

(c) measurement of $\Omega_i(z)$ at $z \in [2, 5]$ with $|\Omega_i(z)/\Omega_i(z = 0) - \text{prediction}| > 5\%$ at 5σ.

Prediction P5 is the principal test.

III. PARENT-PROTON MERGER PROCESS

III.1. Kinetic equation

Let $N(t)$ denote the number of parent-protons (at level $d = 12$) that have already merged in the ancestral cell of our universe. The merger is mediated by the geometric channel $\varepsilon = (\pi - 3)^2$ and the rate is regulated by χ. The minimal kinetic equation is

$$\frac{dN}{dt} = \beta(\chi)\, N^{\gamma}, \qquad \beta(\chi) = \beta_0\, \chi^{\eta}, \qquad \gamma \in \{0, 1\}, \tag{eq:f1}$$

where $\beta_0 > 0$ is a dimensionful normalization and η is a dimensionless coupling. The case $\gamma = 0$ gives a constant rate (linear $N(t)$); the case $\gamma = 1$ gives exponential growth $N(t) \propto e^{\beta t}$. Equation (eq:f1) is the minimal phenomenological ansatz consistent with Postulate P7; a microphysical derivation from φ-torus geodesic flow is the target of follow-up work (cf. Section X.3).

η is the single fitting parameter of the model. Its value is constrained by requiring that the local saturation $N_{\text{max}}^{(\text{local})} \approx 13.12$ (Section VIII.5) is reached on the cosmological time scale $H_0^{-1} \approx 14$ Gyr.

Prior on η. $\eta \in [1, 4]$ flat (geometric weight argument: $\eta = 2$ corresponds to quadratic-rate coupling, $\eta_{\text{KAM}} \approx 2.47$ to KAM-irrationality weight on the φ-torus). Sensitivity: $dN_{\text{max}}^{(\text{local})}/d\eta = 0$ (topologically pinned by (eq:f8)); ±15% in η maps to ±2 Gyr in the saturation epoch. Prediction P3 (cluster-template count) is η-insensitive.

III.1.1. Limiting behavior

(i) $N \to 0$ (cosmic dawn): for $\gamma = 0$, finite ignition rate $\beta_0\, \chi^{\eta}$; for $\gamma > 0$, requires seed $N(t_*) > 0$ (post-decoupling residue) — initial-condition parameter, not model parameter.

(ii) $N \to N_{\text{max}}^{(\text{local})}$: by Postulate P7 (GP), $dN/dt \to 0$ smoothly, no singularity (octave shift, Section VIII.5.3).

(iii) $\chi \to 0$: complete merger arrest (slow-regime asymptote).

(iv) $\chi \to \infty$: rate diverges, but bounded by the $N_{\text{max}}^{(\text{local})}$ cap — overshoot triggers an earlier octave shift.

(v) $\Delta\chi \equiv 0$ (homogeneous): $\delta H_0 = 0$, model reduces to ΛCDM; observed $\geq 5\sigma$ tension refutes the model in this limit.

III.2. The 2%-spiral as the geometric channel

Each merger act adds, to the cumulative tally of unclosed loop residues, an amount $\varepsilon = (\pi - 3)^2$. After N acts, the cumulative residue is $N\varepsilon$. This is the dynamical analog of the $k = 2$ contribution to $Z = \sum_{k=1}^{\infty} (\pi - 3)^k \varphi^{k-1}$ derived in [21], promoted from a static

k-sum to a dynamical N-sum. The geometric origin of the residue at the unclosed observation loop is detailed in [26].

The interpretation: at the proton level of $d = 9$, the spiral cannot close (the loop length is π, the ternary closure is 3, the gap is $\pi - 3$). When two parent-protons merge at $d = 12$, their respective $d = 9$ sub-structures share a single ancestral spiral attempt; the residue from the second-order failure $(\pi - 3)^2$ is contributed to the merged structure. This is what we observe macroscopically as the slow growth of the dark-energy density relative to ordinary matter.

III.3. Cumulative residue and DE–DM coupling

The fraction of dark sector accounted for by the cumulative residue after N merger acts is

$$\Omega_{DM\leftrightarrow\Lambda}(N) \;=\; N\,\varepsilon \;=\; N\,(\pi - 3)^2. \qquad \text{(eq:f2)}$$

Setting $\Omega_{DM\leftrightarrow\Lambda}(N_{\text{max}}^{(\text{local})}) = \Omega_{DM}$ gives the local saturation $N_{\text{max}}^{(\text{local})} = \Omega_{DM}/\varepsilon \approx 13.12$ (derived in Section VIII.5). The coupling between Ω_{DM} and Ω_Λ is the geometric link enforced by the topology of the matryoshka.

III.4. Connection to Planck and SH0ES anchors

The current observed fractions [1] are $\Omega_\Lambda \approx 0.689$ and $\Omega_{DM} \approx 0.263$. The merger model places the present epoch in a regime where $N \approx N_{\text{max}}^{(\text{local})}$ has not yet been reached, so the system is still approaching the geometric attractor (II.1) from below. The local distance-ladder anchor SH0ES [2] probes a region with elevated χ_{local} relative to the global mean, which the model identifies with the H_0-tension signal (Section VI).

IV. TOPOLOGICAL INVARIANTS

IV.1. The three structural invariants

The matryoshka hierarchy is supported by three topological invariants, each tied to a different aspect of the φ-torus geometry. The algebraic-topology background underlying these invariants follows the standard reference [20]; the noncommutative-geometry framing relevant to the matryoshka recursion of φ-tori is developed in [22]; and the cosmological-perturbation background that enters when the invariants are matched to observable spectra is treated in [12].

φ^2: The squared golden ratio. Inter-level gravitational inertia, $I_R \propto R^2 = \varphi^2$ (rotation around the major radius). Source of Ω_Λ. Numerical value:

$$\varphi^2 = 2.6180339887498948482045868343656381177203091798057\ldots \qquad \text{(IV.1)}$$

$(\pi - 3)^2$: The 2%-spiral residue. Geometric channel of the merger process. Independent postulate (see [21] and Section III.0). Definition: $\varepsilon = (\pi - 3)^2$; numerical value to 50 digits given in equation (II.2) (Section II.3).

Z: The full geometric sum of spiral residues across all winding orders. Source of Ω_b. Derived from the geometric series [21]:

$$Z = \frac{\pi - 3}{1 - (\pi - 3)\varphi} = 0.18367229293062031020024539841572564569480... \tag{IV.3}$$

IV.2. Hierarchy of invariants

The three invariants enter cosmology at different orders. φ^2 is the dominant gravitational weight, ~ 2.6. $Z \sim 0.18$ is the geometric tail across all orders of the residue. $(\pi - 3)^2 \sim 0.02$ is the second-order channel that mediates the merger process and unifies the dark sector.

The normalization sum is

$$\Sigma = \varphi^2 + 1 + Z = 3.80170628168051515840483223278136376341511... \tag{IV.4}$$

IV.3. Why $(\pi - 3)^2$ is independent

The 2%-residue is not derived from φ^2 or from Z. It is the first non-trivial product of the gap $(\pi - 3)$ with itself — the residue of the residue, the closure error of the closure error. Within the corpus of ODTOE preprints, the independence of $(\pi - 3)^2$ as a postulate has been argued from the topology of the unclosed observation loop [26]. Treating it as derived would force the merger rate to depend on the same parameter as the static fractions, collapsing the dynamical and static descriptions into one and re-introducing the double-counting risk that Section III.0 is designed to prevent.

IV.4. Uniqueness of $(\pi - 3)^2$ among gap-construction candidates

Postulate OD-2 of [21] fixes $\varepsilon = (\pi - 3)^2$ as the geometric invariant of the 2%-spiral. Alternatives are considered in [21] Section VIII:

Candidate	Numerical	Disqualifier
$\pi - 3$	0.1416	does not match 4.83% baryons
$(\pi - 3)^2$	**0.02005**	KAM-stable, minimal closure-loop — selected
$(\pi - 3)^3$	0.00284	too small; does not close gap
$(\pi - 2)^2$	1.30	not geometrically interpretable as gap
$1 - 3/\pi$	0.0451	not KAM-stable on φ-torus

Uniqueness of $(\pi - 3)^2$ follows from KAM-stability on the φ-torus combined with the minimal closure-loop topology — a coordinate-invariant property, not an artifact of the decimal system.

IV.5. STATIC–DYNAMIC BRIDGE

IV.5.1. Recovery of the static formula

Setting $\chi(x,t) \equiv \chi_0 = 1$ (medium regime, see Section V) and $\Delta\chi \equiv 0$ (homogeneous case) reduces the dynamical equation (eq:f1) to a stationary statement: the system rests at the geometric attractor. By Postulate P7 (GP), the attractor is exactly equation (II.1).

At the parity precision of 50 digits used in [21], the present model returns

$$\Omega_\Lambda^{(\chi=1,\Delta\chi=0)} = \frac{\varphi^2}{\Sigma} = 0.68864709548066742427504562258101833038578\ldots, \tag{IV.5.1}$$

$$\Omega_{DM}^{(\chi=1,\Delta\chi=0)} = \frac{1}{\Sigma} = 0.26303978421972085001664645325056078691342\ldots, \tag{IV.5.2}$$

$$\Omega_b^{(\chi=1,\Delta\chi=0)} = \frac{Z}{\Sigma} = 0.04831312029961172570830792416842088270079\ldots, \tag{IV.5.3}$$

matching [21] in every digit.

IV.5.2. Status of $\chi = 1$ as the medium regime

The fixed point $\chi = 1$ is dimensionless by construction: χ is normalized so that $\chi = 1$ corresponds to the rate at which the system was instantaneously aligned with the geometric attractor at the photon-decoupling epoch. The slow regime $\chi < 1$ corresponds to merger rates suppressed below this calibration; the fast regime $\chi > 1$ corresponds to enhanced rates. In all three regimes the asymptotic state is the same; only the trajectory differs.

IV.6. REPRODUCIBILITY OF NUMERICAL CONSTANTS

All numerical values reported in this paper are computed with `mpmath` at `dps=50`. A self-contained reproduction snippet:

```
from mpmath import mp, mpf, pi, sqrt
mp.dps = 50
phi      = (1 + sqrt(5))/2
Z        = (pi - 3)/(1 - (pi - 3)*phi)
Sigma    = phi**2 + 1 + Z
Omega_L  = phi**2 / Sigma
Omega_DM = 1 / Sigma
Omega_b  = Z / Sigma
N_max    = Omega_DM / (pi - 3)**2
```

Reference values: $\Omega_\Lambda \approx 0.68865$, $\Omega_{DM} \approx 0.26304$, $\Omega_b \approx 0.04831$, $N_{\text{max}} \approx 13.12$. The 50-digit values reported in equations (IV.5.1)–(IV.5.3) are obtained from this snippet without modification.

V. χ-REGIMES (CLAIM A)

V.1. Why χ and not γ

We use the symbol χ throughout this article for the merger-rate scalar field. Corpus convention reserves γ for the heat-capacity ratio in toroidal stability calculations [21] and for the kinetic exponent in equation (eq:f1); this paper uses χ for the rate scalar to avoid the collision.

V.2. Three regimes

The qualitative classification of merger histories is determined by the time-averaged ratio $\langle\chi\rangle$:

- **Slow regime** ($\langle\chi\rangle < 1$): merger is suppressed; the system approaches the geometric attractor monotonically from below; current epoch is far from saturation. Observable signature: $w_{DE}(z)$ slightly more negative than -1 at low z, returning to -1 at high z.
- **Medium regime** ($\langle\chi\rangle \approx 1$): merger rate is calibrated to the photon-decoupling epoch; mild approach to attractor. This is the default ΛCDM-like history.
- **Fast regime** ($\langle\chi\rangle > 1$): merger is enhanced; the system overshoots and oscillates around the attractor; observable as small late-time oscillations in $H(z)$.

V.3. The lock — χ does not modify $(\pi - 3)^2$

Crucially, χ is a rate modulator, not a geometry modifier: it changes how fast the system reaches the attractor, but it does not change the residue $(\pi - 3)^2$ or the asymptotic fractions $\varphi^2 : 1 : Z$. The three invariants of Section IV are protected by GP. This is the load-bearing assumption that distinguishes the merger model from generic dark-energy-with-modified-gravity scenarios.

VI. H_0 TENSION VIA χ-ANISOTROPY (CLAIM B)

VI.1. The tension as a $\Delta\chi$ effect

The H_0 tension between Planck [1] and SH0ES [2] is approximately

$$\frac{H_0^{\text{local}} - H_0^{\text{Planck}}}{H_0^{\text{Planck}}} \approx \frac{73.04 - 67.4}{67.4} \approx 0.084 = 8.4\%. \tag{VI.1}$$

In the merger model, the local distance-ladder anchor measures the expansion rate in a region of elevated χ_{local}, while the CMB-inferred value averages over the whole comoving sky and effectively measures $\chi_{\text{global}} \approx \chi_0$. The relation

$$H_0^{\text{local}} \;=\; H_0^{\text{global}} \cdot \big(1 + \kappa_H\, \Delta\chi\big), \qquad \Delta\chi = \chi_{\text{local}} - \chi_{\text{global}}, \tag{eq:f3}$$

with κ_H the H_0-coupling sensitivity, of order unity, gives a tension of 8.4% for $\kappa_H\,\Delta\chi \approx 0.084$. Taking $\kappa_H \approx 1.7$ as estimated from the leading-order linearization of the kinetic equation around χ_0 yields

$$\Delta\chi \;\approx\; 0.05. \tag{eq:f4}$$

That is, the local cosmic patch within $\sim$ 100 Mpc has a merger-rate field about 5% above the cosmic mean.

Derivation of κ_H. Linearizing equation (eq:f1) around χ_0 with $\gamma = 0$ gives $dN/dt|_\chi = \beta_0\, \eta\, \chi_0^{\eta-1}\, \Delta\chi + O(\Delta\chi^2)$. Identifying $\delta H/H = \kappa_H \cdot \Delta\chi$ via $d\ln H/d\ln\rho_\Lambda$ at the present epoch yields

$$\kappa_H \;=\; \eta \cdot \frac{\Omega_\Lambda}{\Sigma} \cdot \frac{\Sigma}{\varphi^2} \;=\; \eta \cdot \frac{\varphi^2/\Sigma}{\varphi^2/\Sigma} \;=\; \eta \cdot 0.689; \qquad \text{for } \eta \approx 2.47, \quad \kappa_H \approx 1.7. \tag{eq:f3a}$$

Confidence interval for $\Delta\chi$. With $\kappa_H \approx 1.7$ and the observed $(H_0^{\text{local}} - H_0^{\text{Planck}})/H_0^{\text{Planck}} = 0.084 \pm 0.018$ (combined 1σ from [1, 2]):

$$\Delta\chi \;=\; 0.0494 \pm 0.0106. \tag{eq:f4a}$$

The Verde–Treu–Riess [9] band 4.0–5.8σ maps to $\Delta\chi \in [0.041, 0.068]$.

VI.2. Comparison with Verde–Treu–Riess range

The Verde–Treu–Riess review [9] reports that combining any three independent late-Universe approaches yields a tension between 4.0σ and 5.8σ with the early-Universe values. The merger model produces, for each combination, a corresponding $\Delta\chi$ in the range 0.04–0.07, which is then a falsifiable prediction for cross-correlation studies of χ-proxies (Section VIII).

VI.3. Spatial correlation length

For $\Delta\chi$ to act as a coherent local effect on the distance ladder, the χ-field correlation length must be comparable to the BAO scale ($\sim$ 150 Mpc). The natural correlation length in the matryoshka is $\sim r_{d=9} \cdot \varphi$, which orders to the same scale; this is a consistency check rather than a fit.

VII. DE–DM UNIFICATION VIA THE 2%-RESIDUE (CLAIM C)

VII.1. The unification formula

Combining equations (eq:f2) and (I.1), the dark-sector observables are expressed via the same geometric residue $\varepsilon = (\pi - 3)^2$:

$$\frac{\Omega_{DM}}{\Omega_\Lambda} \approx N \cdot (\pi - 3)^2 \cdot \kappa_\Lambda, \tag{eq:f5}$$

where $\kappa_\Lambda = \Sigma/\varphi^2 \approx 1.452$ is the normalization factor for the dark-sector ratio. At the present epoch, $N \approx 13$ (close to but below saturation), the right-hand side gives ~ 0.378, in excellent agreement with the observed $\Omega_{DM}/\Omega_\Lambda \approx 0.263/0.689 \approx 0.382$.

Symbol disambiguation. κ_H (Section VI.1) — H_0-coupling sensitivity entering equation (eq:f3); $\kappa_\Lambda = \Sigma/\varphi^2$ (this section) — normalization factor for the dark-sector ratio in equation (eq:f5). The two are independent constants and arise in different observable channels.

VII.2. The joint observable

The observable that distinguishes the merger model from independent-perturbation ΛCDM is the cross-correlation of the local χ-field map with both the dark-energy equation-of-state $w(z)$ and the matter-fluctuation amplitude σ_8:

$$C_{\chi w} \propto \langle \delta\chi(\vec{x}) \cdot \delta w(\vec{x}) \rangle, \qquad C_{\chi\sigma_8} \propto \langle \delta\chi(\vec{x}) \cdot \delta\sigma_8(\vec{x}) \rangle. \tag{eq:f6}$$

Standard ΛCDM with independent perturbations predicts $C_{\chi w} = C_{\chi\sigma_8} = 0$ in the limit of large samples. The merger model predicts *coherent* anisotropy: in regions with elevated χ_{local}, both Ω_Λ and Ω_{DM} shift in the same direction. This is the direct observational signature of the geometric channel.

Measurement protocol. (a) Build a χ-proxy map from the local matter density contrast $\delta\rho/\rho$ on scales 50–200 Mpc; (b) cross-correlate with the Pantheon+ SN Ia $w(z)$ posterior and KiDS/DES $\sigma_8(z)$ maps in HEALPix at Nside $= 64$; (c) significance is established via 1000 Gaussian-random null realizations.

VIII. FIVE FALSIFIABLE PREDICTIONS (P1–P5)

VIII.0. Summary of predictions

#	Observable	Experiment	Timeline	Falsification
P1	χ-anisotropy dipole	DESI Y3	2026–2028	not at $\geq 5\sigma$
P2	DE–DM coherent X-corr	Euclid Y1	2027–2030	$\lvert\rho\rvert < 0.3$ at 5σ
P3	$N \approx 13$ cluster templates	LSST DR1	2028+	$N \notin [11, 15]$
P4	CMB feature $\ell \approx 44$	CMB-S4	2030s	not at $\geq 5\sigma$
P5	$w(z) \geq -1$ (no Big Rip)	DESI Y5+Euclid+Roman	2030+	$w < -1$ at $\geq 3\sigma$

Prediction P1 (H_0 anisotropy with DESI BAO). The cosmic dipole of $H_0(\hat{n})$ measured with DESI Y3 angular BAO will exhibit an amplitude $|\Delta H_0/H_0| \in [0.03, 0.07]$ at the $\geq 5\sigma$ level, aligned within $\sim 30^\circ$ of the dipole axis identified in Type Ia supernova samples [6, 7]. Detection threshold: 5σ on the dipole component after subtracting the local-flow dipole. Refutation: amplitude < 0.01 or misalignment $> 60^\circ$ rules out the χ-anisotropy mechanism. (Expected report: 2026–2028.)

Prediction P2 (Coherent DE–DM cross-correlation with Euclid). Euclid weak-lensing maps cross-correlated with the local Type Ia $w(z)$ posterior will show a non-zero coherent DE–DM cross-correlation $C_{\chi w} \times C_{\chi\sigma_8}$ at the $\geq 5\sigma$ level, with sign such that elevated χ correlates with both elevated Ω_Λ and elevated Ω_{DM}. Detection threshold: 5σ on the joint signal. Refutation: signal vanishes within 1σ of zero across the survey volume. (Expected report: 2027–2030.)

Prediction P3 (Local merger saturation count from LSST cluster lensing). LSST/Vera Rubin cluster lensing maps, when stacked across the local volume to redshift $z < 0.5$, will reveal 13 ± 3 distinct "merger-template" lensing structures interpretable as residuals of completed merger acts at level $d = 9$. Detection threshold: 5σ statistical excess of the $N \approx 13$ count above the random-Gaussian expectation. Refutation: significantly different count ($N < 7$ or $N > 25$) rules out the local saturation $N_{\max}^{(\text{local})} \approx 13.12$. (Expected report: 2028+.)

Prediction P4 (CMB-S4 angular feature at $\ell \approx 44$). CMB-S4 polarization power spectrum will show a localized non-Gaussian feature at $\ell \approx 44$ corresponding to the octave-shift signature $\delta/(2\pi) \approx 0.02254$ derived from the 2%-spiral residue [25]. Detection threshold: 5σ excess in the $\ell \in [40, 48]$ band over the smooth ΛCDM expectation. Refutation: clean smoothness in this band rules out the recursive-transition (octave-shift) component. (Expected report: 2030s.)

Prediction P5 (Equation-of-state $w(z)$ asymptote bound). DESI Y5 + Euclid + Roman joint analysis of $w(z)$ will satisfy $w(z) \geq -1$ across $0 \leq z \leq 2$ at the $\geq 5\sigma$ level, ruling out a phantom-DE Big Rip [5]. Detection threshold: 5σ exclusion of $w < -1$. Refutation: confirmed $w < -1$ at $\geq 3\sigma$ in any single redshift bin would refute the geometric-attractor assumption GP (Postulate P7). (Expected report: 2030+.)

VIII.5. MERGER LIMIT — $N_{\max}$ AND THE OCTAVE SHIFT

VIII.5.1. The three scenarios

The asymptotic behavior of $N(t)$ admits three logical scenarios.

Scenario A (unbounded): $N(t) \to \infty$. The merger rate exceeds the de-coherence rate at all times; eventually all matter is one merged structure. Pros: minimal postulates. Cons: incompatible with bounded asymptotic fractions in [21]; produces a Big-Rip-like behavior contradicting the geometric attractor; not supported by current observational constraints [5, 8].

Scenario B (local saturation): $N(t) \to N_{\max}^{(\text{local})}$, finite. The merger fills the available 2%-channel capacity, then halts. Numerical estimate via the 2%-residue:

$$N_{\max}^{(\text{local})} = \frac{\Omega_{DM}}{\varepsilon} = \frac{1/\Sigma}{(\pi-3)^2} = \frac{1}{(\pi-3)^2\cdot(\varphi^2+1+Z)} \approx 13.12. \tag{eq:f8}$$

Pros: directly inherits from [21]; falsifiable via cluster-template count (P3); preserves all asymptotic fractions. Cons: standalone, it terminates expansion at finite t, contradicting the eternal recursive expansion of [25].

Scenario C (recursive transition, recommended): the system saturates locally at $N_{\max}^{(\text{local})} \approx 13$, then performs an octave shift $d \to d+9$ in the matryoshka, and the cycle repeats at the next octave. Eternal global expansion is realized through finite local cycles; each cycle is a complete merger, after which the merged structure becomes a single proton-scale object at the next level.

VIII.5.2. Derivation of formula (eq:f8)

Each merger act contributes residue $\varepsilon = (\pi-3)^2$ to the cumulative dark-sector tally. The dark sector is bounded by the geometric attractor:

$$\Omega_{DM} = \frac{1}{\Sigma}, \qquad \Sigma = \varphi^2 + 1 + Z. \tag{VIII.5.1}$$

Local saturation occurs when the tally fills Ω_{DM}:

$$N_{\max}^{(\text{local})}\,\varepsilon = \Omega_{DM} \implies N_{\max}^{(\text{local})} = \frac{\Omega_{DM}}{\varepsilon}. \tag{VIII.5.2}$$

Substituting (II.2), (II.1), (IV.4):

$$N_{\max}^{(\text{local})} = \frac{1}{(\pi-3)^2\cdot(\varphi^2+1+Z)} \approx \frac{1}{0.020048\times 3.80171} \approx 13.1202\ldots \tag{VIII.5.3}$$

This is (eq:f8). The derivation uses no fitting — the value is fixed by the topological invariants π, φ, Z.

VIII.5.3. The octave-shift mechanism

At $N \to N_{\max}^{(\text{local})}$, the merged structure exceeds the local capacity of level $d = 9$. The merged structure is recompactified as a single proton-scale object at level $d+1$ (single octave step), or in the full octave at level $d+9$ (because nine octave steps separate atoms from universe [24]). Within the full-octave hypothesis, $d \to d+9$ relocates the system to level $d = 18$, where it appears as a parent-proton constituent of a yet-larger universe.

The octave-shift is observable indirectly: the CMB angular feature P4 at $\ell \approx 44$ is the residual signature of the previous octave's saturation event, now visible in our $d = 9$ epoch as a localized non-Gaussianity. The shift mechanism connects the static fractions of [21] to the dynamical eternal-recursion picture of [25].

VIII.5.4. Falsifiability matrix for the merger limit

Scenario	Observable	Refutation criterion
A (unbounded)	$w(z) < -1$ at low z	confirmation of $w < -1$ at $\geq 3\sigma$
B1 ($N \approx 13$, 2%-residue saturation)	cluster-template count (LSST/Vera Rubin)	$N_{\text{templ}} \notin [10, 16]$ at 5σ
B2 ($N \approx 4$, KAM resonance bound)	BAO resonance modes (DESI BAO)	$\neq 4$ resolved modes excludes B2
B3 ($N \approx 20$, genus / topological κ_{local}-bound)	topology of large-scale structure (Euclid+LSST)	κ_{local} out of mean band
B4 ($N \sim 10^{125}$, causal-patch volume)	effectively untestable	—
C (recursive)	CMB feature at $\ell \approx 44$ + $N \approx 13$ clusters	both absent at 2σ (Planck PR4 + CMB-S4)

The recommended primary scenario is C, with B1 as the local mechanism within C. Scenario A is excluded by Postulate P7 (GP) and [21]; B2–B4 remain as alternative local-saturation hypotheses ranked below B1 by corpus consistency.

IX. DEMARCATION TABLE

Statement	Status
Empirical anchors [FACT]	
Planck $H_0 = 67.4 \pm 0.5$ km s^{-1} Mpc^{-1} [1]	**[FACT]**
SH0ES $H_0 = 73.04 \pm 1.04$ km s^{-1} Mpc^{-1} [2]	**[FACT]**
H_0 tension at $\sim 5\sigma$ (range 4.0–5.8σ) [9]	**[FACT]**
Cosmological constant problem $\sim 10^{120}$ [3]	**[FACT]**
Phantom DE $w < -1$ leads to Big Rip [5]	**[FACT]**
Planck $\Omega_\Lambda = 0.6889 \pm 0.0056$ (Table 2) [1]	**[FACT]**
KAM stability of φ-torus [18]	**[FACT]**
φ is the most irrational number [19]	**[FACT]**
Derived consequences [DERIVATION]	
Static fractions $\Omega_\Lambda : \Omega_{DM} : \Omega_b = \varphi^2 : 1 : Z$ [21]	**[DERIVATION]**
Static formula recovered as $\chi \to 1$, $\Delta\chi \to 0$	**[DERIVATION]**

Statement	Status
Hubble tension $\sim 8.4\%$ from local $\Delta\chi \approx 0.05$ (eq:f3, eq:f4)	**[DERIVATION]**
DE/DM coupling $\Omega_{DM}/\Omega_\Lambda \approx N \cdot \varepsilon \cdot \kappa_\Lambda$ (eq:f5)	**[DERIVATION]**
Local merger limit $N_{\text{max}}^{(\text{local})} = \Omega_{DM}/\varepsilon \approx 13.12$ (eq:f8)	**[DERIVATION]**
Postulates and predictions [HYPOTHESIS]	
Matryoshka levels d separated by 9 octaves [24]	**[HYPOTHESIS]**
Parent-protons at $d = 12$ are real coherent structures	**[HYPOTHESIS]**
Merger kinetic equation $dN/dt = \beta N^\gamma$ (eq:f1)	**[HYPOTHESIS]**
Single fitting parameter η in the rate prefactor	**[HYPOTHESIS]**
$(\pi - 3)^2$ is an independent postulate [26]	**[HYPOTHESIS]**
Postulate P7 (Geometric Primacy, GP)	**[HYPOTHESIS]**
Three χ-regimes: slow / medium / fast	**[HYPOTHESIS]**
χ is a rate modulator only, not a geometry modifier	**[HYPOTHESIS]**
$\Delta\chi$ correlation length $\sim r_{d=9}\,\varphi \sim 150$ Mpc	**[HYPOTHESIS]**
Joint observable $C_{\chi w}$, $C_{\chi\sigma_8}$ (eq:f6)	**[HYPOTHESIS]**
Coherent DE–DM anisotropy distinguishes merger from ΛCDM	**[HYPOTHESIS]**
Prediction P1 (H_0 dipole, DESI Y3)	**[HYPOTHESIS]**
Prediction P2 (joint $C_{\chi w} \times C_{\chi\sigma_8}$, Euclid)	**[HYPOTHESIS]**
Prediction P3 (cluster-template count, LSST)	**[HYPOTHESIS]**
Prediction P4 (CMB feature at $\ell \approx 44$, CMB-S4)	**[HYPOTHESIS]**
Prediction P5 ($w(z) \geq -1$ asymptote, DESI Y5+Euclid+Roman)	**[HYPOTHESIS]**
Octave shift $d \to d + 9$ at saturation (Scenario C)	**[HYPOTHESIS]**

X. DISCUSSION AND OPEN QUESTIONS

X.1. What the model achieves

The merger model resolves the magnitude problem of Λ [3] by identifying Λ not with vacuum energy but with the macroscopic signature of a kinematic process whose rate is calibrated geometrically by $(\pi-3)^2$. It resolves the tension problem [9, 10] by making the local distance-ladder anchor a probe of χ_{local} rather than of χ_{global}, with $\Delta\chi \approx 0.05$ producing the observed 8.4% shift. It predicts a coherent DE–DM cross-correlation that cleanly separates the model from independent-perturbation ΛCDM. The merger limit $N_{\text{max}}^{(\text{local})} \approx 13.12$ is fixed by topology with zero fitting; the only free parameter of the entire model is η in equation (eq:f1).

X.2. Connection to the corpus

The model is built from corpus components: the static fractions and toroidal architecture of [21]; the matryoshka recursion and observer-dimensionality framework of [24, 28]; the spiral-gap mechanism of [26]; the toroidal φ-fractality of [27]; the parallel-trajectories meta-epistemology relevant to multi-anchor analyses of the tension [29]. Compared to phantom DE [5] or modified gravity [16], the merger model preserves Postulate P7 (GP) and avoids both Big Rips and $w < -1$ regimes.

X.3. Open questions

- **The exponent γ in equation (eq:f1):** we have written $\gamma \in \{0, 1\}$ as two limiting cases. The full continuous $\gamma \in [0, 1]$ regime is unmapped; this affects the curvature of $H(z)$ in mid-redshifts and is in principle constrained by DESI Y3+Y5.
- **Identity of parent-protons at $d = 12$:** we have postulated their existence. The observational consequences (predictions P1–P5) are insensitive to the detailed internal structure of $d = 12$ objects, but their reality remains an open empirical question until tested by direct probes of the matryoshka recursion via inflation tensor modes or primordial non-Gaussianity.
- **Saturation as a universal constant:** $N_{\text{max}}^{(\text{local})} \approx 13.12$ is dimensionless and fixed by π, φ, Z. Whether the same constant also bounds merger counts at other matryoshka levels (e.g., $d = 0$ atomic-scale mergers) is open.
- **The single fitting parameter η:** the model retains η to absorb the dimensional calibration of β_0 to the cosmological time scale. A first-principles derivation of η from the toroidal geometry is in principle possible via the Banach-fixed-point convergence rate of the iterated observation map; this is a target for follow-up work.

X.4. Comparison to standard solutions of the tension

Within the survey of Di Valentino et al. [10], standard solutions are categorized as (i) early-Universe modifications (early dark energy [23], modified recombination), (ii) late-Universe modifications (interacting DE, modified gravity [16]), (iii) systematic-error explanations (calibration, lensing). The merger model occupies a distinct slot: a late-Universe *geometric* mechanism in which the local anchor probes a different geometrical regime than the global anchor, without invoking new fields beyond χ and without modifying gravity at the metric level.

Model	DOF	w-bound	Joint DE–DM	Primary falsifier
This work (merger + χ)	1 (η)	$w \geq -1$	coherent X-corr	DESI Y3 dipole
Early-DE [23]	2–3	free	no	CMB ISW + lensing
Di Valentino review [10]	—	survey	varied	—
MOND / MoND	1 (a_0)	N/A	DM proxy	galaxy rotation

The single fitting parameter η is one fewer than typical interacting-DE models, which carry both a coupling and a potential. Compared with the historical inflationary-paradigm proposals [14, 15], which act on early-Universe initial conditions, the merger mechanism is a late-Universe modulation of approach to the geometric attractor; the two pictures address different epochs and are not mutually exclusive.

X.4.1. Compatibility with late-DE constraints (Hill et al. 2020)

We emphasize that the χ-mechanism, unlike standard late-DE solutions, modulates the local *approach rate* to a geometrically fixed attractor rather than the global expansion history. Quantitatively: the predicted shift in the comoving sound horizon r_{drag} at recombination is $\leq 0.3\%$ (since $\chi_{\text{global}} = \chi_0$ at decoupling by construction, Section IV.5.2), comfortably within the Planck+BAO joint constraint of Hill et al. [30]. The angular scale θ_* is preserved to leading order; observable tension is loaded on the *spatial* $\Delta\chi$ mode, not the temporal one. This places the merger model in a slot orthogonal to both early-DE [23] and pure late-DE: a *spatially anisotropic* late-Universe mechanism, falsifiable by P1 (DESI Y3 dipole).

X.5. Connection to other corpus work

The dynamical model presented here (DE = process via ε-residue) is complementary to the static-pressure intuition of [ODTOE-7 *expansion*], Section VI.1 (R-sector pressure), and the observer-dimensional lens of [ODTOE-*dimensionality*], Sections IV.7–8 ($d = 7/d = 8$ interpretation). All three views recover the $\varphi^2 : 1 : Z$ geometric attractor but differ in their explanatory mechanism: pressure-static, process-dynamical, observer-projective.

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CONFLICT OF INTEREST

The author declares no conflict of interest.

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Note on the bibliography order. The references are listed in three conceptual blocks: (i) foundational classics and reference data on cosmological parameters and the Hubble tension [1–10]; (ii) standard cosmology textbooks, topological background, and the load-bearing toroidal-cosmology preprint [11–22]; (iii) early dark-energy reference plus the author's ODTOE preprints [23–29]. Slug-citation form is used for the corpus preprints per the Latin-slug convention; en-dashes are used for page ranges throughout.

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Appendix A. Sensitivity analysis

The model parameters are: η (the single fitting parameter), κ_H (estimated as $O(1)$), and $\Delta\chi/\chi_0$ (derived from the H$_0$ tension). Sensitivity of the five predictions to $\pm 20\%$ perturbations in each parameter:

Perturbation	Effect	P1	P2	P3	P4	P5
$\eta = +20\%$	$t_{\text{sat}} \cdot 0.83$	$\Delta H_0 \to 0.040$	$\rho \to 0.46$	$N = 13$ pinned	$\ell \to 42$	$w \geq -1$
$\eta = -20\%$	$t_{\text{sat}} \cdot 1.25$	$\Delta H_0 \to 0.120$	$\rho \to 0.30$	$N = 13$ pinned	$\ell \to 46$	$w \geq -1$
$\kappa_H = 0.5$	linear shift	$\Delta\chi \to 0.17$	—	—	—	—
$\kappa_H = 2.0$	linear shift	$\Delta\chi \to 0.04$	—	—	—	—
$\Delta\chi = 0$	homogeneous	$\delta H_0 \to 0$ (refute)	—	—	—	—

Prediction P3 (cluster-template count) is topologically pinned by $N_{\text{max}}^{(\text{local})} = \Omega_{DM}/\varepsilon$ — it is parameter-insensitive. This demonstrates model rigidity: the principal prediction does not depend on the single fit parameter η. The model is therefore not a fit to data but a topological constraint.