COSMOLOGICAL FRACTIONS FROM TOROIDAL ARCHITECTURE: DERIVING THE CONTENT OF DARK ENERGY, DARK MATTER AND BARYONIC MATTER FROM π AND φ

Pankratov Anton Sergeevich

Independent researcher, Kazan, Russia

E-mail: anton.s.pankratov@gmail.com

ORCID: 0009-0002-4870-2995

UDC 524.8 + 530.12 + 514.7 + 167.7

ABSTRACT

Within the toroidal model of ODTOE, the cosmological fractions of dark energy, dark matter and baryonic (visible) matter content of the Universe are derived from two structural invariants: π and φ. The φ-torus (a torus with radii ratio $R/r = \varphi$, maximally stable by the KAM theorem) possesses three topological sectors: the inter-level sector (major radius R, gravitational inertia $\propto R^2 = \varphi^2$), the intra-level sector (minor radius r, inertia $\propto r^2 = 1$), and the gap sector (accumulated spiral gaps, full series $Z = (\pi-3)/[1-(\pi-3)\varphi]$). Normalized fractions: $\Omega_\Lambda : \Omega_{DM} : \Omega_b = \varphi^2 : 1 : Z = 68.86\% : 26.30\% : 4.83\%$. Comparison with Planck 2018 data (TT,TE,EE+lowE+lensing) [1]: $68.47\pm0.73\%$ (dark energy), $26.07\pm0.73\%$ (cold dark matter Ω_c), $4.93\pm0.06\%$ (baryonic). Dark energy and dark matter fall *within* the 1σ Planck confidence interval (0.54σ and 0.32σ respectively). Baryonic matter deviates by 1.64σ (within 2σ). A self-referential correction (by analogy with the formulas for μ and α^{-1} [10]) improves the agreement to 1.24σ. The formula contains zero adjustable parameters. All three components are expressed through π and φ, which arise from the Banach theorem [17] as the continuous and discrete invariants of convergence to the fixed point.

Keywords: dark energy, dark matter, baryonic matter, cosmological fractions, ODTOE, φ-torus, KAM theorem, spiral gap, number π, golden ratio.

I. INTRODUCTION

I.1. The Problem

The observable Universe consists of three main components by energy density: dark energy ($\Omega_\Lambda \approx 68.5\%$), dark matter ($\Omega_{DM} \approx 26.5\%$), baryonic matter ($\Omega_b \approx 5\%$) [1]. The standard model of cosmology (ΛCDM) accepts these fractions as empirical parameters fitted to data from the cosmic microwave background, Type Ia supernovae and baryon acoustic oscillations [18, 19]. The question of *why* these particular fractions — remains open. The cosmological constant Λ is not derived from first principles; the discrepancy between the prediction of quantum field theory and observation amounts to $\sim 10^{120}$ (the cosmological constant problem [2]).

I.2. The Approach

ODTOE [3] models reality as a hierarchy of nested φ-tori [4]: each level of dimensionality d is represented by a torus with radii ratio $R/r = \varphi$, maximally stable by the KAM theorem [5, 6, 7]. The three topological sectors of the torus give rise to three components of cosmological content. Below it is shown that the normalized sector fractions coincide with Planck 2018 data [1] within 1σ–2σ.

This work uses results from the ODTOE series of papers: the toroidal topology of reality [4], the structure of the number π as an invariant of observation [9], the derivation of the fundamental constants μ and α^{-1} [10], the model of the atom as a strange loop [11], the architecture of the quantum [14], the dimensionality of the observer [15] and Planck's constant from the architecture of observation [16].

II. THE φ-TORUS: THREE TOPOLOGICAL SECTORS

II.1. Definition

A torus with major radius R and minor radius r, $R/r = \varphi = (1+\sqrt{5})/2$. The trajectory on the torus is described by two angular coordinates: θ (rotation around the minor radius, fast) and ϕ (rotation around the major radius, slow). By the KAM theorem [5, 6, 7]: when the frequency ratio $\omega_\theta/\omega_\phi = R/r = \varphi$ (the most irrational number [8]) the torus is maximally stable against perturbations. The trajectory is quasi-periodic: it never closes, densely filling the surface.

II.2. Three Sectors

Sector I: inter-level (R-dynamics). Rotation along the major radius = transition between dimensionality levels d. Associated with macrostructure: expansion of the Universe, cosmological constant. Via ODTOE: pressure of the field $\mathcal{H}$ (infinite) on the finite configuration $\mathcal{C}$.

Sector II: intra-level (r-dynamics). Rotation along the minor radius = phase dynamics within a single level d. Associated with structure formation: gravitational binding, halo formation. Via ODTOE: coherent configurations at levels $d > d_{\text{our}}$, invisible by D-Prot but gravitating by P5 [3].

Sector III: gap ($(\pi-3)$-dynamics). Accumulated spiral gaps: each revolution along θ does not close (length $= \pi > 3$, gap $= \pi - 3$), generating a remainder. The sum of remainders from all windings = visible matter. Via ODTOE: everything that is born in the gap of the observation loop — photons, atoms, stars, observers [9, 14].

II.3. Gravitational Inertia of the Sectors

Each sector contributes to the total gravitational inertia of the Universe. Here *gravitational inertia* denotes the contribution of the corresponding mode to the T^{00} component of the energy-momentum tensor [20]. The fraction Ω_i is determined by the gravitational weight of the corresponding degree of freedom. For rotational motion the gravitational weight is proportional to the moment of inertia:

$$I = m \cdot r_{\text{eff}}^2 \tag{II.1}$$

For R-rotation: $I_R = mR^2$. For r-rotation: $I_r = mr^2$. The ratio:

$$\frac{I_R}{I_r} = \frac{R^2}{r^2} = \varphi^2 \tag{II.2}$$

Justification: in general relativity, the contribution of a component to the total energy density is determined by the energy-momentum tensor $T_{\mu\nu}$ [20]. For a perfect fluid: $T^{00} = \rho c^2$ (energy density). For rotational motion the kinetic energy density $\propto I\omega^2/V$. But by the KAM condition $\omega_\theta/\omega_\phi = R/r = \varphi$, whence $I_R\omega_\phi^2/(I_r\omega_\theta^2) = (R^2/r^2)\times(\omega_\phi/\omega_\theta)^2 = \varphi^2/\varphi^2 = 1$. The kinetic energies are *equal* (the virial theorem for a KAM torus).

However, *gravitational inertia* is determined not by kinetic energy, but by the *total* energy (kinetic + potential + pressure). For the cosmological constant: $p = -\rho c^2$ (negative pressure), the contribution to the effective gravitational mass $\propto \rho + 3p/c^2 = -2\rho$ [2, 20]. It is precisely this *anomalous* contribution (through pressure) that scales as R^2, not $R^2\omega^2$. The total effective mass of the sector:

$$M_{\text{eff},\,R} \propto R^2, \quad M_{\text{eff},\,r} \propto r^2 \tag{II.3}$$

The ratio of *gravitational weights* = $\varphi^2 : 1$ — is determined by the geometry of the torus, not by the dynamics.

III. CONTRIBUTION OF THE GAP SECTOR

III.1. One Winding

Each revolution along the minor radius (θ) does not close: the path length $= \pi$, the minimum closed path $= 3$ (ternary architecture [9]). The first-order gap:

$$\delta_1 = \pi - 3 = 0.14159265358979... \tag{III.1}$$

III.2. Full Spiral Series

Each winding generates a gap scaled by φ (the step between windings on the torus). The k-th order gap: $(\pi-3)^k \cdot \varphi^{k-1}$. The full series:

$$Z = \sum_{k=1}^{\infty} (\pi-3)^k \cdot \varphi^{k-1} = \frac{(\pi-3)}{1-(\pi-3)\varphi} \tag{III.2}$$

Convergence: the ratio $(\pi-3)\varphi = 0.22910... < 1$. The series is geometric.

Numerical value (50 digits):

$$Z = 0.18367229293062031020024539841572564569480... \tag{III.3}$$

Decomposition by contributions:

Order k	Contribution $(\pi-3)^k\varphi^{k-1}$	Fraction of Z
1	0.14159	77.1%
2	0.03244	17.7%
3	0.00743	4.0%
4	0.00170	0.9%
5+	0.00051	0.3%

Visible matter consists of 77% from the "first winding" ($k=1$) and 23% from higher orders. The higher orders are critical for agreement with Planck: without them $\Omega_b = 3.77\%$ (first order), with them $\Omega_b = 4.83\%$ (full series).

IV. NORMALIZED FRACTIONS

IV.1. Formula

$$\Omega_\Lambda : \Omega_{DM} : \Omega_b = \varphi^2 : 1 : Z \tag{IV.1}$$

Normalization:

$$\Sigma = \varphi^2 + 1 + Z \tag{IV.2}$$

$$\Omega_\Lambda = \frac{\varphi^2}{\Sigma}, \quad \Omega_{DM} = \frac{1}{\Sigma}, \quad \Omega_b = \frac{Z}{\Sigma} \tag{IV.3}$$

IV.2. Numerical Values (50 digits for reproducibility)

Note. 50 digits are provided for exact reproducibility of computations; the precision of Planck data is $\pm 0.73\%$ ($\sim$3 significant figures).

$$\varphi^2 = 2.6180339887498948482045868343656381177203 1...$$

$$Z = 0.18367229293062031020024539841572564569480...$$

$$\Sigma = 3.80170628168051515840483223278136376341511...$$

$$\Omega_\Lambda = 0.68864709548066742427504562258101833038578... = 68.865\%$$

$$\Omega_{DM} = 0.26303978421972085001664645325056078691342... = 26.304\%$$

$$\Omega_b = 0.04831312029961172570830792416842088270080... = 4.831\%$$

Check: $\Omega_\Lambda + \Omega_{DM} + \Omega_b = 1.00000000000000000000000000000000000000000$.

V. COMPARISON WITH PLANCK 2018 DATA

V.1. Experimental Values

Planck mission data [1] (TT,TE,EE+lowE+lensing, 68% CL):

$$\Omega_\Lambda = 0.6847 \pm 0.0073 \tag{V.1}$$

$$\Omega_c = 0.2607 \pm 0.0073 \quad \text{(cold dark matter)} \tag{V.2}$$

$$\Omega_b = 0.0493 \pm 0.0006 \quad \text{(baryonic matter)} \tag{V.3}$$

Additionally: $\Omega_\nu \approx 0.0014$ (neutrinos, at $\sum m_\nu = 0.06$ eV), $\Omega_r \approx 0.0001$ (cosmic microwave background radiation).

V.2. Identification of Components

ODTOE sector	Cosmological component	Mechanism (ODTOE)
φ^2 (inter-level)	Ω_Λ (dark energy)	Pressure of $\mathcal{H}$ on $\mathcal{C}$ via R
1 (intra-level)	Ω_c (cold dark matter)	Coherent structures at $d > d_{\text{our}}$
Z (gap)	Ω_b (baryonic)	Matter from the spiral gap

Neutrinos (Ω_ν) and radiation (Ω_r) are not separated into distinct sectors: their contributions $\sim 0.15\%$ are absorbed by the main components. In the four-component model (Section VII) neutrinos are separated out.

V.3. Comparison Table

Component	ODTOE, %	Planck 2018, %	Dev., %	σ
Dark energy (Ω_Λ)	**68.86**	68.47 ± 0.73	$+0.39$	**0.54**
Dark matter (Ω_{DM})	**26.30**	26.07 ± 0.73	$+0.23$	**0.32**
Baryonic (Ω_b)	**4.83**	4.93 ± 0.06	-0.10	**1.64**

Dark energy and dark matter: *within* the 1σ confidence interval. Baryonic: *within* 2σ, deviation 1.64σ.

VI. SELF-REFERENTIAL CORRECTION

VI.1. Justification

The formulas for $\mu = m_p/m_e$ and α^{-1} [10] contain self-referential terms: the proton mass enters its own definition ($(\pi-3)^2/\mu$), the fine-structure constant enters its own equation. For cosmological fractions: the baryonic matter fraction *affects* the total gravitational dynamics, which *determines* the conditions for the existence of baryons. The loop $\Omega_b \leftrightarrow$ conditions for baryon creation [11].

VI.2. Quadratic Equation

Denote $x = \Omega_b$, $\varepsilon = (\pi-3)^2$, $K = \varphi^2 + 1$:

$$x = \frac{Z + \varepsilon x}{K + Z + \varepsilon x} \tag{VI.1}$$

Expanding:

$$\varepsilon x^2 + x(K + Z - \varepsilon) - Z = 0 \tag{VI.2}$$

Solution (positive root):

$$x = \frac{-(K + Z - \varepsilon) + \sqrt{(K + Z - \varepsilon)^2 + 4\varepsilon Z}}{2\varepsilon} \tag{VI.3}$$

Numerical values:

$$a = \varepsilon = 0.02004847955...$$

$$b = K + Z - \varepsilon = 3.78165780213...$$

$$c = -Z = -0.18367229293...$$

$$D = b^2 + 4ac = 14.31566513325...$$

$$\sqrt{D} = 3.78360478027...$$

$$x = \Omega_b^{(\mathrm{sr})} = 0.04855675290... = 4.856\%$$

VI.3. Recalculation of All Fractions

$$\Omega_b^{(\mathrm{sr})} = 4.856\%$$

$$\Omega_\Lambda^{(\mathrm{sr})} = \frac{\varphi^2}{\varphi^2+1}\left(1 - \Omega_b^{(\mathrm{sr})}\right) = 68.847\%$$

$$\Omega_{DM}^{(\mathrm{sr})} = \frac{1}{\varphi^2+1}\left(1 - \Omega_b^{(\mathrm{sr})}\right) = 26.297\%$$

VI.4. Comparison (with Self-Reference)

Component	Without self-ref.	With self-ref.	Planck 2018	σ (self-ref.)
Ω_Λ	68.86%	**68.85%**	68.47 ± 0.73	**0.52**
Ω_{DM}	26.30%	**26.30%**	26.07 ± 0.73	**0.31**
Ω_b	4.83%	**4.86%**	4.93 ± 0.06	**1.24**

The self-referential correction improves the agreement for baryons: $1.64\sigma \to 1.24\sigma$.

VII. FOUR-COMPONENT MODEL (WITH NEUTRINOS)

VII.1. Neutrinos as the Second-Order Gap

Via ODTOE: neutrinos = spiral remainder of the observation loop [11, Section IV.3]. Their contribution = $(\pi - 3)^2 = 0.02005$ (squared gap, second order):

$$\Sigma_4 = \varphi^2 + 1 + Z + (\pi - 3)^2 = 3.82175... \tag{VII.1}$$

$$\Omega_\Lambda^{(4)} = 68.50\%, \quad \Omega_{DM}^{(4)} = 26.17\%, \quad \Omega_b^{(4)} = 4.81\%, \quad \Omega_\nu^{(4)} = 0.52\% \tag{VII.2}$$

VII.2. Comparison

Planck gives $\Omega_\nu \approx 0.14\%$ (at the minimum mass sum $\sum m_\nu = 0.06$ eV) [1]. The ODTOE model: 0.52%. Discrepancy: $\times 3.7$. Significant. However: the Planck upper limit on $\sum m_\nu$ is < 0.12 eV (95% CL), which gives $\Omega_\nu < 0.27\%$. At $\sum m_\nu \approx 0.15$ eV (admissible in extended models [21]): $\Omega_\nu \approx 0.34\%$, closer to 0.52%.

Status: the four-component model *predicts* a neutrino mass sum *higher* than the minimum. This is a falsifiable prediction in tension with the current Planck upper limit ($\Omega_\nu < 0.27\%$ at 95% CL). If future data (KATRIN, DESI, CMB-S4) confirm $\Omega_\nu < 0.3\%$, the four-component model will be refuted. The three-component model (Sections IV–VI) retains its validity.

VIII. BINARY AND TERNARY φ-PROPORTIONS

VIII.1. Binary (Work/Rest)

With two components (without the gap): $\varphi/(1+\varphi) : 1/(1+\varphi) = 61.8\% : 38.2\%$. Observed in optimal work/rest regimes (62:38), inhalation/exhalation, systole/diastole [12].

VIII.2. Ternary (Universe)

With three components (with the gap): $\varphi^2 : 1 : Z = 68.9\% : 26.3\% : 4.8\%$. Observed in the composition of the Universe [1].

VIII.3. Connection

Dark energy in the ternary model (68.9%) is *greater* than in the binary model (61.8%), because the gap sector (4.8%) is a small third contribution, and its "share" is mostly compensated by the major radius. The binary φ-proportion is the *limit* of the ternary one as $Z \to 0$ (the gap tends to zero, $\pi \to 3$):

$$\lim_{\pi\to 3} \frac{\varphi^2}{\varphi^2+1+Z} = \frac{\varphi^2}{\varphi^2+1} = \frac{\varphi}{1+\varphi} = 61.8\% \qquad \text{(VIII.1)}$$

The ternary proportion *reduces* to the binary one in the limit of zero gap. $\pi > 3$ is the reason why the cosmological fractions *differ* from the "pure" φ-proportion.

IX. DEMARCATION

Statement	Status
KAM theorem: the φ-torus is maximally stable	**Proven** [5, 6, 7]
φ is the most irrational number	**Proven** [8]
Three sectors of the torus $\to$ three components	**Interpretation** via ODTOE
Gravitational inertia $\propto R^2 : r^2 = \varphi^2 : 1$	**Follows** from $T_{\mu\nu}$ for torus modes
Full series $Z = (\pi - 3)/(1 - (\pi - 3)\varphi)$	**Follows** from the geom. series of gaps
$\Omega_\Lambda = 68.86\%$ (within 1σ of Planck)	**Numerical result**, zero fitting
$\Omega_{DM} = 26.30\%$ (within 1σ of Planck)	**Numerical result**, zero fitting
$\Omega_b = 4.83\%$ (1.64σ from Planck)	**Numerical result**, refinement
Self-ref. correction: $\Omega_b = 4.86\%$ (1.24σ)	**Follows** by analogy [10]
Neutrinos $= (\pi - 3)^2$, $\Omega_\nu \approx 0.52\%$	**Falsifiable** prediction
Binary $\to$ ternary at $\pi > 3$	**Mathematical fact** (limit)

X. CONCLUSION

X.1. Result

From two numbers (π and φ) and one geometric construction (the φ-torus), three cosmological fractions are derived without adjustable parameters:

$$\boxed{\Omega_\Lambda : \Omega_{DM} : \Omega_b = \frac{\varphi^2}{\varphi^2 + 1 + Z} : \frac{1}{\varphi^2 + 1 + Z} : \frac{Z}{\varphi^2 + 1 + Z}} \tag{X.1}$$

$$\text{where } Z = \frac{(\pi - 3)}{1 - (\pi - 3)\varphi} \tag{X.2}$$

$$= 68.86\% : 26.30\% : 4.83\% \tag{X.3}$$

Agreement with Planck 2018: 0.54σ, 0.32σ, 1.64σ (all within 2σ, two out of three within 1σ).

X.2. Structure of the Formula

φ^2 = gravitational weight of inter-level dynamics (dark energy = pressure of $\mathcal{H}$ on $\mathcal{C}$) [3, 4].

1 = gravitational weight of intra-level dynamics (dark matter = coherent structures at $d > 3$) [3, 15].

$Z = (\pi - 3)/(1 - (\pi - 3)\varphi)$ = accumulated gaps of all windings (visible matter = everything born in the gap) [9, 14].

X.3. What This Means

The Universe consists of $\sim 95\%$ "torus" (φ^2+1: two rotations, invisible to us) and $\sim 5\%$ "gap" (Z: what is born each time the loop fails to close). We are the gap. We are $(\pi-3)$, multiplied by φ and summed over all windings of the spiral. A small fraction, but the *only visible one*. The remaining 95% is the torus on which we live but which we do not see, as a fish does not see the water.

$$\boxed{\text{We} = \frac{(\pi-3)}{1-(\pi-3)\varphi} = 4.83\% \text{ of the Universe} = \text{sum of all spiral gaps}}$$

ACKNOWLEDGEMENTS AND TOOLS

During the development of the ODTOE theory and all papers based on it, artificial intelligence tools were used: Claude Sonnet / Opus 4.6 Extended (Chat & Code) (Anthropic), ChatGPT 5.3 (OpenAI), Google Gemini (Google DeepMind). All substantive decisions, hypotheses, interpretations and responsibility for them belong to the author.

CONFLICT OF INTEREST

The author declares no conflict of interest.

FUNDING

This work was carried out without external funding.

REFERENCES

[1] Planck Collaboration. Planck 2018 results. VI. Cosmological parameters // Astronomy & Astrophysics. — 2020. — Vol. 641. — Art. A6. DOI: 10.1051/0004-6361/201833910. arXiv: 1807.06209.

[2] Weinberg S. The Cosmological Constant Problem // Reviews of Modern Physics. — 1989. — Vol. 61(1). — P. 1–23.

[3] Pankratov A. S. Theory of Everything: Observer-Dependent (ODTOE) // Preprint. — 2025. — 47 p.

[4] Pankratov A. S. Toroidal Topology of Reality: Nested φ-Tori // Preprint. — 2026.

[5] Kolmogorov A. N. On the Preservation of Conditionally Periodic Motions under Small Changes of the Hamiltonian Function // Doklady Akad. Nauk SSSR. — 1954. — Vol. 98. — P. 527–530.

[6] Arnold V. I. Small Denominators and Problems of Stability of Motion in Classical and Celestial Mechanics // Uspekhi Mat. Nauk. — 1963. — Vol. 18(6). — P. 91–192.

[7] Moser J. On Invariant Curves of Area-Preserving Mappings of an Annulus // Nachr. Akad. Wiss. Gottingen, Math.-Phys. Kl. II. — 1962. — P. 1–20.

[8] Khinchin A. Ya. Continued Fractions. — Chicago: University of Chicago Press, 1964.

[9] Pankratov A. S. The Number π as a Structural Invariant of Self-Consistent Observation // Preprint. — 2025.

[10] Pankratov A. S. Two Fundamental Constants from First Principles: μ and α^{-1} // Preprint. — 2026.

[11] Pankratov A. S. The Atom as an Elementary Strange Loop in ODTOE // Preprint. — 2025.

[12] Pankratov A. S. Extracting Energy from the Field of Potential States // Preprint. — 2026.

[13] Riess A. G. et al. A Comprehensive Measurement of the Local Value of the Hubble Constant // The Astrophysical Journal Letters. — 2022. — Vol. 934(1). — Art. L7.

[14] Pankratov A. S. Architecture of the Quantum: π, φ and the Spiral Gap // Preprint. — 2026.

[15] Pankratov A. S. Dimensionality of the Observer and the Octaves of Reality // Preprint. — 2026.

[16] Pankratov A. S. Planck's Constant from the Architecture of Observation // Preprint. — 2026.

[17] Banach S. Sur les opérations dans les ensembles abstraits et leur application aux équations intégrales // Fundamenta Mathematicae. — 1922. — Vol. 3. — P. 133–181.

[18] Perlmutter S. et al. Measurements of Ω and Λ from 42 High-Redshift Supernovae // The Astrophysical Journal. — 1999. — Vol. 517(2). — P. 565–586.

[19] Eisenstein D. J. et al. Detection of the Baryon Acoustic Peak in the Large-Scale Correlation Function of SDSS Luminous Red Galaxies // The Astrophysical Journal. — 2005. — Vol. 633(2). — P. 560–574.

[20] Misner C. W., Thorne K. S., Wheeler J. A. Gravitation. — San Francisco: W. H. Freeman, 1973. — 1279 p.

[21] Lesgourgues J., Pastor S. Massive neutrinos and cosmology // Physics Reports. — 2006. — Vol. 429(6). — P. 307–379.