COHERENCE AS A MEASURABLE QUANTITY: THREE CONSEQUENCES OF THE HURST EXPONENT — S PARAMETER RELATION FOR THE ODTOE FORMALISM

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ABSTRACT

It is established that the relation between the Hurst exponent of fractional Brownian motion and the ODTOE coherence parameter ($H = (1 + S)/2$ [1]) gives rise to three consequences for the theory's formalism.

First: coherence S becomes independently measurable via the anomalous-diffusion exponent α determined from the mean-square displacement ($S = \alpha - 1$). This renders all ODTOE predictions containing S experimentally testable.

Second: substituting $S = \alpha - 1$ into the Planck constant formula $h(d, S) = 2\pi(\pi - 3)^2\varphi^{d+1}\Sigma(d)(1 - S)^{-1/2}A_0$ [2] yields a dependence of h on the diffusion exponent: $h \propto (2-\alpha)^{-1/2}$, predicting a deviation of the effective quantum of action in highly coherent systems (BEC, superconductors).

Third: the dimensionless parameter $r = R_0^2(\pi - 3)^2\varphi^d/[2D_0(1 - S)\tau_0]$ [1], governing the transition from the stochastic to the drift regime, quantitatively describes the strengthening of the arrow of time with increasing observation scale. The critical dimensionality level at which gap drift suppresses stochasticity ($r = 1$ at $S = 0$) is $d_{\text{crit}} \approx 8.12$, coinciding with the metagalactic level ($d = 8$) in the ODTOE observation hierarchy.

Keywords: coherence, measurability, Planck constant, arrow of time, anomalous diffusion, Hurst exponent, ODTOE, spiral gap.

I. INTRODUCTION

I.1. The measurability problem of coherence

Coherence S is the central parameter of the Observer-Dependent Theory of Everything (ODTOE), governing the transition from the quantum ($S \to 0$) to the classical ($S \to 1$) regime [3, formula 4.4a]. Prior to the present work, S was defined exclusively through the internal metric of the observer cluster [3, formula 4.5]:

$$S = 1 - \frac{2}{n(n-1)} \sum_{i<j} |B_i - B_j| \tag{I.1}$$

Formula (I.1) requires knowledge of the individual contextual belief values B_i for each observer in the cluster. For atomic ($d = 0$) and subatomic ($d < 0$) observers, direct measurement of B_i is experimentally inaccessible. Consequently, all ODTOE predictions containing S remained unfalsifiable at the microscale until an alternative determination path was established.

Note the boundary behaviour of formula (I.1). For $n = 2$ it reduces to $S = 1 - |B_1 - B_2|$. Full coherence ($S = 1$) is reached when $B_1 = B_2$, i.e. when contextual beliefs are identical. Full decoherence ($S = 0$) requires $|B_1 - B_2| = 1$, i.e. maximal disagreement. For arbitrary n, the quantity S represents a normalised measure of cluster unanimity.

I.2. The $H(S)$ relation and its consequences

In [1] it was established that the Hurst exponent of fractional Brownian motion (fBm) is related to coherence by:

$$H(S) = \frac{1+S}{2} \tag{I.2}$$

The anomalous-diffusion exponent is defined through the mean-square displacement (MSD):

$$\langle x^2(\tau) \rangle \propto \tau^{\alpha} \tag{I.3}$$

The relation $\alpha = 2H$ is a standard result of fractional Brownian motion theory [17]. Substituting (I.2):

$$\alpha = 2H = 2 \cdot \frac{1+S}{2} = 1 + S \tag{I.4}$$

from which the key identity of the present work follows:

$$\boxed{S = \alpha - 1} \tag{I.5}$$

The exponent α is measured by standard condensed-matter techniques: correlation analysis of density fluctuations, single-particle tracking, neutron reflectometry [4, 5]. Formula (I.5) transforms coherence S from a theoretical construct into a physical quantity with a concrete experimental determination procedure.

I.3. Domain of values and consistency

Formula (I.5) imposes constraints on admissible values. ODTOE coherence is defined on the interval $S \in [0, 1)$ [3]. Substituting the boundaries:

$$S = 0 \;\Rightarrow\; \alpha = 1 \quad \text{(normal diffusion)}, \tag{I.6}$$

$$S \to 1 \;\Rightarrow\; \alpha \to 2 \quad \text{(ballistic regime)}. \tag{I.7}$$

Normal diffusion ($\alpha = 1$) corresponds to zero coherence — a fully stochastic regime. Ballistic transport ($\alpha = 2$) corresponds to maximal coherence. Subdiffusion ($\alpha < 1$) is excluded in this model: negative values of S are not defined within the ODTOE formalism. This is consistent with the fact that coherence describes the degree of observer agreement and is non-negative by definition.

The Hurst exponent $H = (1 + S)/2$ correspondingly takes values $H \in [1/2, 1)$. The value $H = 1/2$ is standard Brownian motion (Markov process). Values $H > 1/2$ describe a persistent process with long-range memory. The anti-persistent domain ($H < 1/2$) is excluded, which is physically meaningful: in ODTOE, observers build coherence that reinforces correlations but does not suppress them below the Markov baseline.

I.4. Structure of the paper

The present work develops three consequences of formula (I.5). Section II addresses independent measurement of S via anomalous diffusion. Section III establishes the dependence of the Planck constant on the diffusion exponent. Section IV provides a quantitative description of the arrow of time through the parameter r. Section V links the three consequences into a unified chain. Section VI contains a demarcation table. Section VII is the conclusion.

II. CONSEQUENCE 1: INDEPENDENT MEASUREMENT OF S

II.1. Two methods of determining coherence

Prior to the present work, the only path to determining S was the cluster internal metric, formula (I.1). Formula (I.5) opens a second, diffusion-based method. A comparison of the two approaches is given in Table 1.

Table 1: Comparison of two methods for determining coherence S

Characteristic	**Method 1 (internal)**	**Method 2 (diffusion)**
Formula	$S = 1 - \frac{2}{n(n-1)} \sum_{i<j} \lvert B_i - B_j \rvert$	$S = \alpha - 1$
Measured quantity	Individual values of B_i	Slope of $\log \mathrm{MSD}$ vs $\log \tau$
Applicability	Systems with known B_i (groups of people, collectives)	Any system with observable diffusion

Characteristic	Method 1 (internal)	Method 2 (diffusion)
Limitation	Inaccessible for atomic observers	Requires sufficiently long trajectories
Uncertainty	Determined by the B_i scale precision	Standard error of linear regression

The existence of two independent methods for determining the same quantity creates an opportunity for cross-validation: if both methods yield coincident values of S for the same system within experimental uncertainty, this confirms the internal consistency of the ODTOE formalism.

II.2. Mathematical basis for cross-validation

Let S_1 denote the coherence obtained from formula (I.1) and $S_2 = \alpha - 1$ the value obtained from a diffusion measurement. The ODTOE formalism predicts:

$$S_1 = S_2 \pm \delta, \tag{II.1}$$

where δ is determined by the experimental uncertainties of both methods. Let σ_1 be the uncertainty in S_1 (depending on the B_i scale precision and the sample size n), and σ_2 the uncertainty in α (depending on trajectory length and temporal window). The discrepancy threshold is then:

$$\delta = \sqrt{\sigma_1^2 + \sigma_2^2} \tag{II.2}$$

Falsifiable prediction: $|S_1 - S_2| < 3\delta$ for 99.7% of measurements (three-sigma criterion). Systematic disagreement $S_1 \neq S_2$ beyond 3δ would refute either formula (I.1) or the relation $H(S) = (1 + S)/2$.

II.3. Experimental protocol for cross-validation

For a system of n observers with measurable B_i (a group of people, a choir, a sports team), the following protocol is proposed:

Step 1. Measure B_i for each participant through the components F, E, σ, Λ [3, formula D1.1]. Minimum group size $n \geq 5$ for statistical significance.

Step 2. Compute S_1 from formula (I.1). Estimate the uncertainty σ_1 via bootstrap (recomputing S upon exclusion of one participant).

Step 3. In parallel, record a time series of the group's joint activity over at least 10^3 characteristic time scales τ_0. For a human group, $\tau_0 \sim 1$ s; hence the minimum recording duration is 10^3 s (≈ 17 min).

Step 4. Compute the MSD from formula (I.3) for time lags τ from τ_0 to $10^2\tau_0$. Determine α as the slope of the linear regression of $\log\langle x^2\rangle$ versus $\log\tau$. Estimate σ_2 as the standard error of the slope coefficient.

Step 5. Compute $S_2 = \alpha - 1$.

Step 6. Verify $|S_1 - S_2| < 3\delta$ using formula (II.2).

II.4. Catalogue of measurable systems

The relation $S = \alpha - 1$ makes it possible to determine coherence for systems where direct measurement of B_i is inaccessible. Table 2 summarises systems in which the exponent α has already been measured but was not previously interpreted as a measure of coherence.

Table 2: Systems with measured anomalous-diffusion exponent

System	d	**Method of measuring** α	**Existing data**
Ions in plasma	0	Correlation spectroscopy	[6]
Proteins in cell	0–1	Single-particle tracking (SPT)	[7, 8]
Cells in tissue	1	Migration microscopy	[9]
Neurons	2	EEG/fMRI time-series analysis	[10]
Group of people	3–4	Variability of joint activity	Proposed in the present work
Atoms in BEC	0	Expansion interferometry	[11]

For each row of Table 2, the exponent α has already been measured in published studies. Retrospective analysis of these data allows extraction of S values for dozens of experimental systems without additional experiments.

II.5. Estimated S values for specific systems

Based on published values of α, preliminary estimates of coherence can be given:

Table 3: Expected coherence values for experimental systems

System	α (meas.)	$S = \alpha - 1$	**Source of** α
Lipid granules in vivo	1.2 ± 0.1	0.2 ± 0.1	[7]
Chromosomal loci	0.39	Excluded ($S < 0$)	[8]
Amoeboid migration	1.3 ± 0.15	0.3 ± 0.15	[9]
Neuronal oscillations	1.1 ± 0.05	0.1 ± 0.05	[10]
BEC (ballistic)	≈ 2.0	≈ 1.0	[11, 12]

The case of chromosomal loci ($\alpha \approx 0.39$ [8]) warrants separate comment. Subdiffusion ($\alpha < 1$) yields negative S, which lies outside the domain of ODTOE coherence. This indicates that the formula $S = \alpha - 1$ is applicable only to superdiffusive and normally diffusive regimes. Subdiffusion describes systems with anti-persistent correlations and requires separate treatment within an extended formalism [16].

III. CONSEQUENCE 2: PLANCK CONSTANT AS A FUNCTION OF THE DIFFUSION EXPONENT

III.1. Substituting $S = \alpha - 1$ into the h formula

The Planck constant formula derived in [2]:

$$h(d,\, S) = 2\pi(\pi - 3)^2\, \varphi^{d+1}\, \Sigma(d)\, (1 - S)^{-1/2}\, A_0 \tag{III.1}$$

Substituting $S = \alpha - 1$ from (I.5):

$$1 - S = 1 - (\alpha - 1) = 2 - \alpha \tag{III.2}$$

$$h(d,\, \alpha) = 2\pi(\pi - 3)^2\, \varphi^{d+1}\, \Sigma(d)\, (2 - \alpha)^{-1/2}\, A_0 \tag{III.3}$$

Define the structural coefficient, which depends only on the dimensionality level:

$$K(d) = 2\pi(\pi - 3)^2\, \varphi^{d+1}\, \Sigma(d) \tag{III.4}$$

Then formula (III.3) takes the compact form:

$$\boxed{h(d,\, \alpha) = K(d)\, (2 - \alpha)^{-1/2}\, A_0} \tag{III.5}$$

III.2. Computing $K(3)$ with precision control

At $d = 3$ (human observer) the components of $K(3)$ are computed from fundamental constants.

Intermediate quantities at elevated precision:

$$\pi - 3 = 0.14159265358979323846264338327950\ldots \tag{III.6a}$$

$$(\pi - 3)^2 = 0.02004847955059918805863070019913\ldots \tag{III.6b}$$

$$\varphi = \frac{1+\sqrt{5}}{2} = 1.61803398874989484820458683436564\ldots \tag{III.6c}$$

$$\varphi^4 = 6.85410196624968454461376050309691\ldots \tag{III.6d}$$

The structural sum $\Sigma(3) = 1.05539$ was computed in [2].

Hence:

$$K(3) = 2\pi \times 0.020048 \times 6.85410 \times 1.05539 = 0.91122 \tag{III.7}$$

Step-by-step verification:

$$2\pi \times (\pi - 3)^2 = 6.28318 \times 0.020048 = 0.12598 \tag{III.7a}$$

$$0.12598 \times \varphi^4 = 0.12598 \times 6.85410 = 0.86326 \tag{III.7b}$$

$$0.86326 \times \Sigma(3) = 0.86326 \times 1.05539 = 0.91109 \tag{III.7c}$$

The discrepancy in the fifth digit (0.91122 vs 0.91109) is due to rounding of intermediate factors. Using full precision, $K(3) = 0.91122$.

III.3. Self-consistency at α^*

From the normalisation condition $h(3, S^*) = A_0$ [2], the coherence of the standard measurement medium was previously computed:

$$S^* = 0.16968 \tag{III.8}$$

The corresponding diffusion exponent:

$$\alpha^* = 1 + S^* = 1.16968 \tag{III.9}$$

Physical meaning: the medium in which standard physical measurements are performed is characterised by weak superdiffusion ($\alpha^* \approx 1.17$ instead of the normal $\alpha = 1$). The 17% deviation from normal diffusion is a numerical consequence of formalism self-consistency, not a fitting parameter.

Normalisation check:

$$2 - \alpha^* = 0.83032 \tag{III.10a}$$

$$(0.83032)^{-1/2} = \frac{1}{\sqrt{0.83032}} = 1.09743 \tag{III.10b}$$

$$K(3) \times (2 - \alpha^*)^{-1/2} = 0.91122 \times 1.09743 = 1.00000 \tag{III.10c}$$

Self-consistency holds exactly: $h(3, \alpha^*) = A_0$ confirms the correctness of the substitution.

III.4. Prediction: h depends on the coherence of the medium

If a measurement is performed in a medium with coherence different from S^*, the observed h differs from the standard value. The ratio:

$$\frac{h(\alpha)}{h(\alpha^*)} = \sqrt{\frac{2 - \alpha^*}{2 - \alpha}} = \sqrt{\frac{0.83032}{2 - \alpha}} \tag{III.11}$$

Numerical values are given in Table 4.

Table 4: Dependence of effective h on the diffusion exponent

α	S	$(2 - \alpha)$	$h(\alpha)/h(\alpha^*)$	**System**
1.00	0.00	1.000	0.9112	Isolated particle
1.17	0.17	0.830	1.0000	Standard conditions (α^*)
1.50	0.50	0.500	1.2887	Moderate coherence
1.90	0.90	0.100	2.8815	BEC, superconductor
1.99	0.99	0.010	9.1122	Extreme coherence

Verification of the $\alpha = 1.00$ row:

$$\sqrt{\frac{0.83032}{1.000}} = \sqrt{0.83032} = 0.91122 \quad \checkmark \tag{III.12}$$

Verification of the $\alpha = 1.50$ row:

$$\sqrt{\frac{0.83032}{0.500}} = \sqrt{1.66064} = 1.28866 \approx 1.2887 \quad \checkmark \tag{III.13}$$

Verification of the $\alpha = 1.90$ row:

$$\sqrt{\frac{0.83032}{0.100}} = \sqrt{8.3032} = 2.88152 \approx 2.8815 \quad \checkmark \tag{III.14}$$

III.5. Sensitivity analysis of $h(\alpha)$

The derivative of the ratio $h(\alpha)/h(\alpha^*)$ with respect to α:

$$\frac{\partial}{\partial\alpha}\left[\frac{h(\alpha)}{h(\alpha^*)}\right] = \frac{1}{2}\sqrt{\frac{0.83032}{(2-\alpha)^3}} \tag{III.15}$$

At $\alpha = \alpha^* = 1.17$:

$$\left.\frac{\partial}{\partial\alpha}\right|_{\alpha^*} = \frac{1}{2}\sqrt{\frac{0.83032}{0.83032^3}} = \frac{1}{2\times 0.83032} = 0.6022 \tag{III.16}$$

A change $\Delta\alpha = 0.01$ around α^* shifts h/h^* by $\approx 0.6\%$. At $\alpha = 1.90$ the sensitivity increases:

$$\left.\frac{\partial}{\partial\alpha}\right|_{1.90} = \frac{1}{2}\sqrt{\frac{0.83032}{0.001}} = 14.41 \tag{III.17}$$

Near extreme coherence the sensitivity grows as $(2-\alpha)^{-3/2}$, making measurements in this region experimentally attractive: small changes in α produce measurable shifts in h_{eff}.

III.6. Experimental verification

The prediction is testable in systems with controllable coherence. A Bose–Einstein condensate ($\alpha \approx 2$, $S \approx 1$) exhibits ballistic expansion (MSD $\propto t^2$) [11, 12]. The effective quantum of action in BEC should differ from the standard h.

Specific procedure:

Step 1. Prepare a Bose–Einstein condensate (e.g. ^{87}Rb, $T \approx 100$ nK, $N \sim 10^5$ atoms).

Step 2. Measure the momentum dispersion Δp and position dispersion Δx for atoms in the condensate via time-of-flight imaging.

Step 3. In parallel, perform an analogous measurement for a thermal cloud (same isotope, same density, $T > T_c$).

Step 4. Compute the effective quantum of action from the uncertainty relation:

$$\Delta x \cdot \Delta p \geq \frac{h_{\text{eff}}}{2} \tag{III.18}$$

Step 5. Compare h_{eff}(BEC) and h_{eff}(thermal).

Prediction: $h_{\text{eff}}(\text{BEC}) > h_{\text{eff}}(\text{thermal})$. Numerical estimate at $\alpha_{\text{BEC}} \approx 1.9$: $h_{\text{eff}}/h \approx 2.88$.

III.7. Interpretive limitation

The formula $h(d, S)$ describes the observation grain of an operator with specific d and S [2, Section XV]. The D-Prot assumption [3] guarantees that each observer perceives its own h as absolute. Direct comparison of h at different S requires an observer capable of simultaneously registering both systems. For an observer with $d = 3$ and $S = S^* = 0.17$, the standard h is that observer's own grain. The prediction $h(\alpha)/h(\alpha^*) \neq 1$ is testable only through indirect effects: changes in effective scattering cross-sections, coherence lengths, interference contrasts.

III.8. Comparison with existing approaches

The hypothesis of variability of fundamental constants has a long history in physics (Dirac's hypothesis on variation of the gravitational constant, theories with variable α_{em}). Formula (III.5) differs from these approaches in two respects:

1. h does not change with time — it is determined by the coherence of the medium in which a given measurement is performed.

2. The deviation mechanism is structural rather than cosmological: (III.5) follows from the architecture of observation, not from cosmic expansion or field interactions.

IV. CONSEQUENCE 3: QUANTITATIVE ARROW OF TIME

IV.1. Qualitative result of ODTOE

In [13] it was proved that the arrow of time follows from the transcendence of π. The iteration sequence $\{\Psi_n\}$ is non-periodic because the phase increment θ contains π as a factor and $\pi/(2\pi) = 1/2$ is irrational (Statement T1 [13]). Irreversibility is not postulated — it follows from the arithmetic properties of π.

This result is qualitative: the loop does not close, the arrow exists. However, it does not address the question: why is physics nearly reversible at the atomic scale while irreversibility is manifest at the macroscale?

IV.2. The parameter r as a measure of directionality

The parameter r, introduced in [1], defines the ratio of directed drift (generated by the spiral gap) to stochastic noise:

$$r(d,\, S) = \frac{R_0^2(\pi - 3)^2\, \varphi^d}{2D_0(1 - S)\tau_0} \tag{IV.1}$$

Drift is a manifestation of the arrow of time (unidirectional shift of the distribution centre). Stochasticity is the random component that masks the arrow. When $r \ll 1$ the arrow is buried in noise. When $r \gg 1$ the arrow dominates.

Define the dimensionless arrow strength:

$$\boxed{A(d,\, S) = \frac{r}{1 + r}} \tag{IV.2}$$

The quantity $A \in [0,\ 1)$: at $r = 0$ the arrow is absent (pure stochasticity); as $r \to \infty$ the arrow is absolute ($A \to 1$, but does not reach 1, consistent with $S < 1$ in the ODTOE formalism).

IV.3. Structure analysis of the parameter r

The parameter r factorises as:

$$r(d,\ S) = \underbrace{\frac{R_0^2(\pi - 3)^2}{2D_0\tau_0}}_{r_0} \cdot \varphi^d \cdot \frac{1}{1 - S} \tag{IV.3}$$

where r_0 is a base quantity determined by the fundamental gap parameters (R_0, D_0, τ_0). From Table 5 (row $d = 0$, $S = 0$) it follows that $r_0 = 0.020$, which coincides with $(\pi - 3)^2 = 0.02005$ to within rounding. This is not a fit but a consequence of normalisation: $R_0^2/(2D_0\tau_0) = 1$ in units where the gap scales are matched.

The scaling factor φ^d ensures exponential growth of r with observation level. Each hierarchical level increases the drift-to-noise ratio by a factor of $\varphi \approx 1.618$. The coherence factor $(1 - S)^{-1}$ amplifies r at non-zero coherence, since coherence suppresses the stochastic component.

IV.4. Dependence on observation level

Since $r \propto \varphi^d$, the arrow strength increases monotonically with observation level. Numerical values are given in Table 5.

Table 5: Arrow-of-time strength $A(d,\ S)$ across observation levels

d	**Observer**	r (S=0)	A (S=0)	r (S=0.9)	A (S=0.9)
0	Atom	0.020	0.020	0.200	0.167
3	Human	0.085	0.078	0.849	0.459
6	Star	0.359	0.264	3.589	0.782
8	Metagalaxy	0.940	0.484	9.396	0.904
9	Universe	1.520	0.603	15.203	0.938

Verification of key cells.

Row $d = 3$, $S = 0$:

$$r(3,\ 0) = 0.020 \times \varphi^3 = 0.020 \times 4.2361 = 0.08472 \approx 0.085 \quad \checkmark \tag{IV.4a}$$

$$A(3,\ 0) = \frac{0.085}{1.085} = 0.0783 \approx 0.078 \quad \checkmark \tag{IV.4b}$$

Row $d = 8$, $S = 0.9$:

$$r(8,\, 0.9) = \frac{0.020 \times \varphi^8}{1 - 0.9} = \frac{0.020 \times 46.979}{0.1} = 9.396 \quad \checkmark \tag{IV.4c}$$

$$A(8,\, 0.9) = \frac{9.396}{10.396} = 0.9038 \approx 0.904 \quad \checkmark \tag{IV.4d}$$

At the atomic level ($d = 0$, $S = 0$) the arrow strength is $A = 0.020$ — the arrow constitutes 2% of the stochastic background. This quantitatively explains the approximate reversibility of quantum mechanics: the Schrödinger equation is invariant under time reversal because at $d = 0$ the gap drift is negligible.

At the cosmological level ($d = 9$, $S = 0$) $A = 0.603$ — the arrow dominates. At $S = 0.9$: $A = 0.938$ — irreversibility is nearly absolute.

IV.5. Critical dimensionality level

The condition $r = 1$ at $S = 0$ determines the critical level at which drift and stochasticity are balanced:

$$(\pi - 3)^2 \cdot \varphi^{d_{\text{crit}}} = 1 \tag{IV.5}$$

Solution:

$$\varphi^{d_{\text{crit}}} = \frac{1}{(\pi - 3)^2} = \frac{1}{0.02005} = 49.879 \tag{IV.6}$$

$$d_{\text{crit}} = \frac{\ln(49.879)}{\ln(1.61803)} = \frac{3.9092}{0.4812} = 8.1245 \tag{IV.7}$$

The quantity $d_{\text{crit}} \approx 8.12$ is computed strictly from $(\pi - 3)^2$ and φ without fitting parameters. Rounding up to an integer: $d_{\text{crit}} = 9$. In the ODTOE hierarchy, $d = 8$ is the metagalaxy (large-scale cosmic structure), $d = 9$ is self-observation of the Universe [14, 15]. The arrow of time becomes the dominant dynamical factor precisely at the cosmological scale, where irreversible expansion is observed.

The coincidence of $d_{\text{crit}} = 9$ with the self-observation level $\Psi^* = \Phi(\Psi^*)$ is substantive. Closing the loop at the cosmological scale requires that drift (directionality) dominate over stochasticity (randomness). It is precisely when $r > 1$ that the system acquires a stable direction, necessary for the existence of a fixed point.

IV.6. Sensitivity analysis of d_{crit}

Formula (IV.7) contains two input parameters: $(\pi - 3)^2$ and φ. Both are fundamental mathematical constants that do not admit variation. Nevertheless, it is useful to estimate the sensitivity of d_{crit} to hypothetical deviations in order to confirm the robustness of the result.

Let $(\pi-3)^2 \to (\pi-3)^2(1+\varepsilon)$. Then:

$$\Delta d_{\text{crit}} = -\frac{\varepsilon}{\ln\varphi} \approx -2.08\,\varepsilon \tag{IV.8}$$

A 1% deviation in $(\pi-3)^2$ shifts d_{crit} by ± 0.02 — the result is robust.

Similarly, with $\varphi \to \varphi(1+\eta)$:

$$\Delta d_{\text{crit}} \approx -d_{\text{crit}} \cdot \eta = -8.12\,\eta \tag{IV.9}$$

A 0.1% deviation in φ shifts d_{crit} by ± 0.008. The result is determined by mathematical constants and is in this sense exact.

IV.7. Arrow of time as a function of coherence

At fixed d, the arrow strength increases monotonically with coherence:

$$A(d,\,S) = \frac{r(d,\,S)}{1+r(d,\,S)} \tag{IV.10}$$

Since $r \propto (1-S)^{-1}$, as $S \to 1$: $r \to \infty$, $A \to 1$. The directionality of time is absolute for a fully coherent system. This is consistent with postulate P3 [3]:

$$T(C) = \frac{T_0}{(1-S)^n} \to \infty \quad \text{as} \quad S \to 1 \tag{IV.11}$$

An infinitely coherent configuration persists forever ($A = 1$: irreversibility is absolute, return is impossible).

For an observer with $d = 3$ and $S^* = 0.17$:

$$r(3,\,0.17) = \frac{0.08472}{1-0.17} = \frac{0.08472}{0.83} = 0.10207 \tag{IV.12}$$

$$A(3,\,0.17) = \frac{0.10207}{1.10207} = 0.09262 \tag{IV.13}$$

The arrow strength for a human under standard conditions: $A \approx 9.3\%$. The arrow exists but is comparatively weak — this permits recollection of the past (partial reversibility) and planning of the future (partial directionality), while complete reversal of time is excluded (irreversibility is non-zero).

IV.8. Relation of $A(d,\,S)$ to the thermodynamic arrow

Classical thermodynamics defines the arrow of time through entropy growth: $\Delta S_{\text{th}} \geq 0$. The parameter A describes an arrow of a different origin — the observation arrow, generated by the drift of the spiral gap. The connection between the two arrows is established through the fluctuation-dissipation theorem [18]. The stochastic component of the gap ($\propto 1 - A$) determines the variance of fluctuations, while the drift component ($\propto A$) determines the mean rate of irreversible entropy production:

$$\langle \dot{S}_{\text{th}} \rangle \propto A(d, S) \tag{IV.14}$$

At $d = 0$, $A = 0.02$: the mean entropy production rate is negligible — quantum processes are nearly reversible. At $d = 9$, $A = 0.94$: entropy production dominates — the cosmological arrow is unambiguous.

V. UNIFICATION OF THE THREE CONSEQUENCES

The three consequences are not isolated. They form a closed chain:

Measurability of S (Consequence 1) $\to$ substitution into $h(d, S)$ $\to$ dependence of h on α (Consequence 2).

Measurability of S $\to$ substitution into $r(d, S)$ $\to$ quantitative arrow (Consequence 3).

Consequences 2 and 3 are linked through the h formula: the factor $(1 - S)^{-1/2}$ in h defines the coherence correction to the observation grain, while the parameter r determines how strongly that grain is directed (contains an arrow) as opposed to isotropic (pure noise).

V.1. Limiting regimes

As $\alpha \to 2$ ($S \to 1$):

$$h \to \infty, \quad A \to 1, \quad r \to \infty \tag{V.1}$$

The observation grain is infinitely large — the observer encompasses everything. The arrow is absolute. Drift completely suppresses stochasticity. The three limits are consistent: an absolutely coherent observer possesses an infinite grain, absolute irreversibility, and zero stochasticity.

As $\alpha \to 1$ ($S \to 0$):

$$h \to h_{\text{min}} = K(d)\, A_0, \quad A \to r_0\, \varphi^d \ll 1, \quad r \to r_0\, \varphi^d \tag{V.2}$$

An incoherent observer possesses a minimal grain, reversible dynamics, and maximal stochasticity — the quantum limit.

V.2. Unified linking formula

Combining (III.5) and (IV.1), the effective quantum of action can be expressed through the arrow strength:

$$h = K(d)\, A_0\, (1-S)^{-1/2} = K(d)\, A_0\, \left(\frac{r}{r_0\, \varphi^d}\right)^{1/2} \tag{V.3}$$

As $A \to 1$ ($r \to \infty$): $h \to \infty$. As $A \to 0$ ($r \to 0$): $h \to K(d)\, A_0$. The quantum of action and the arrow of time are two manifestations of a single coherence parameter.

VI. DEMARCATION

Table 6: Epistemic status of assertions

Assertion	Status
$H(S) = (1+S)/2$	Hypothesis, verified numerically [1]
$S = \alpha - 1$	Follows from $H(S)$ and $\alpha = 2H$
$h(d,\, \alpha) = K(d)(2-\alpha)^{-1/2} A_0$	Follows from substituting $S = \alpha - 1$ into [2]
$h(\alpha)/h(\alpha^*) \neq 1$ at $\alpha \neq \alpha^*$	Falsifiable prediction
$A(d,\, S) = r/(1+r)$	Definition; r follows from [1]
$d_{\text{crit}} \approx 8.12$	Computed from $(\pi - 3)^2$ and φ
Agreement of S_1 and S_2	Falsifiable prediction
$\alpha^* = 1.170$ (standard measurement medium)	Follows from $S^* = 0.16968$ [2]

VII. CONCLUSION

The relation $H(S) = (1+S)/2$, established in [1], gives rise to three consequences that strengthen the ODTOE formalism.

The first consequence transforms coherence S from a theoretical construct into a measurable physical quantity ($S = \alpha - 1$), opening a path to experimental verification of all ODTOE formulas containing S. A cross-validation protocol has been developed, requiring simultaneous determination of S from the internal-metric formula and from anomalous diffusion. A catalogue of systems with already measured values of α has been compiled.

The second consequence renders the Planck constant a function of the diffusion exponent ($h \propto (2-\alpha)^{-1/2}$), generating a falsifiable prediction regarding deviation of the effective quantum of action in highly coherent systems. Self-consistency at $\alpha^* = 1.17$ has been verified exactly: $K(3)(2-\alpha^*)^{-1/2} = 1.00000$. Sensitivity analysis shows that at $\alpha > 1.9$ the deviations in h_{eff} are large enough for experimental detection.

The third consequence provides a quantitative description of the strengthening of the arrow of time with observation scale through the parameter r. The critical level

$d_{\text{crit}} \approx 8.12$ is computed from the fundamental constants $(\pi - 3)^2$ and φ and coincides with the cosmological scale ($d = 8$–9), explaining why irreversibility is manifest at the macroscale. For an observer with $d = 3$, the arrow strength is $A \approx 9.3\%$ — an intermediate value that simultaneously permits memory of the past and asymmetry between past and future.

The three consequences are mutually consistent and consistent with previously published ODTOE results. None of them requires additional assumptions: each is derived from the already established formalism through the single relation $H(S) = (1 + S)/2$.

CONFLICT OF INTEREST

The author declares no conflict of interest.

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