BROWNIAN MOTION AS A MANIFESTATION OF OBSERVATIONAL ARCHITECTURE: HURST EXPONENT, COHERENCE, AND THE GOLDEN RATIO SCALING FACTOR φ

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ABSTRACT

Within the framework of the Observer-Dependent Theory of Everything (ODTOE), an interpretation of Brownian motion as a manifestation of observational architecture is proposed. A relation between the Hurst exponent H of fractional Brownian motion and coherence S is established: $H(S) = (1 + S)/2$. The formula reproduces two experimentally confirmed limits: at $S = 0$ (complete decoherence) $H = 1/2$ — classical Brownian motion; at $S = 1$ (complete coherence) $H = 1$ — ballistic determinism. Numerical verification on synthetic fractional Brownian motion trajectories (4096 points, 40 realizations, 9 values of H) shows that the measured MSD exponent $\alpha = 2H$ matches the prediction to within 0.04–$1.54\,\%$. The scaling factor between observation levels equals φ^H, where φ is the golden ratio: at $S = 0$ the ratio of spatial scales of adjacent levels is $\sqrt{\varphi} \approx 1.2720$; at $S = 1$ it is $\varphi \approx 1.6180$. A sixth role of the spiral gap $(\pi - 3)^2$ in ODTOE formalism is identified: the gap determines the parameter r governing the transition from the stochastic (quantum) to the drift (classical) regime. Comparisons with Mandelbrot's fractional Brownian motion theory, the Feynman path integral, fractal analysis of financial markets, and anomalous diffusion in biological systems are presented.

Keywords: Brownian motion, fractional Brownian motion, Hurst exponent, ODTOE, coherence, golden ratio, Hausdorff dimension, anomalous diffusion, spiral gap, path integral.

I. INTRODUCTION

I.1. The problem

Brownian motion, described by Einstein in 1905 [1] and confirmed by Perrin's experiments in 1909 [2], is the random walk of a particle immersed in a fluid. The trajectory is fractal: at any magnification the path remains equally jagged. Mathematically, the process is described by a Wiener process with Hausdorff dimension $d_H = 2$ [3].

The classical theory of Brownian motion rests on the Langevin equation:

$$m\,\ddot{x} = -\gamma\,\dot{x} + \xi(t), \tag{1.1}$$

where m is the particle mass, γ is the friction coefficient, and $\xi(t)$ is the random force with $\langle \xi(t)\rangle = 0$ and $\langle \xi(t)\xi(t')\rangle = 2\gamma k_B T\,\delta(t-t')$. The corresponding Fokker–Planck equation describes the evolution of the distribution function [23]. The Kubo fluctuation–dissipation theorem [24] establishes a general relation between the stochastic force and the dissipation coefficient.

The fractional generalization (Mandelbrot, Van Ness, 1968 [4]) is parametrized by the Hurst exponent $H \in (0,1)$: at $H = 1/2$ — classical Brownian motion (independent increments), at $H > 1/2$ — persistent (positive correlations), at $H < 1/2$ — antipersistent (negative correlations). The mean square displacement (MSD) obeys

$$\langle |x(t+\tau) - x(t)|^2\rangle \sim \tau^{2H}, \tag{1.2}$$

which for $H \neq 1/2$ produces anomalous diffusion [5]. The exponent $\alpha = 2H$ determines the diffusion type: $\alpha < 1$ — subdiffusion, $\alpha = 1$ — normal diffusion, $\alpha > 1$ — superdiffusion, $\alpha = 2$ — ballistic regime.

Fractional Brownian motion $B_H(t)$ is defined via the Mandelbrot–Van Ness stochastic integral [4]:

$$B_H(t) = \frac{1}{\Gamma(H+1/2)}\left[\int_{-\infty}^{0}\left((t-s)^{H-1/2} - (-s)^{H-1/2}\right)dW(s) + \int_{0}^{t}(t-s)^{H-1/2}\,dW(s)\right], \tag{1.3}$$

where $W(s)$ is the standard Wiener process and Γ is the gamma function. The covariance function takes the form

$$\langle B_H(t)\,B_H(s)\rangle = \tfrac{1}{2}\left(|t|^{2H} + |s|^{2H} - |t-s|^{2H}\right). \tag{1.4}$$

Modern single-particle tracking (SPT) experiments in living cells have revealed that the Hurst exponent is not fixed but varies from trajectory to trajectory [6]. Subdiffusion ($H < 1/2$) is observed for chromosomal loci in bacteria [7], lipid granules in yeast [7], and membrane proteins [8]. Superdiffusion ($H > 1/2$) has been recorded for amoeboid motion [9] and motor proteins. This diversity lacks a unified explanation within standard physics.

The problem of classifying anomalous diffusion has attracted considerable attention. Muñoz-Gil et al. [22] carried out a systematic comparison of methods for decoding anomalous diffusion within the AnDi Challenge, establishing that no existing method simultaneously provides reliable estimation of H and classification of the diffusion mechanism. The formula $H(S) = (1+S)/2$ proposed in this work provides a single parameter S that explains the continuous spectrum of H values.

I.2. What ODTOE proposes

In the Observer-Dependent Theory of Everything [10], the reconfiguration dynamics equation contains a stochastic term $\eta(t)$ with variance $D(\eta) = D_0(1-S)$, where S is the coherence of the observer cluster [10, formula 4.4a]. As $S \to 1$ the stochasticity vanishes (determinism, GR). As $S \to S_{\text{min}}$ the stochasticity is maximal (quantum mechanics). A single parameter S governs the transition between the two regimes [10, Section X].

Formally, the stochastic term of the reconfiguration equation reads

$$\frac{\partial \Psi}{\partial t} = \mathcal{F}[\Psi] + \eta(t), \qquad \langle \eta(t)\, \eta(t') \rangle = 2D_0(1 - S)\, \delta(t - t'), \tag{1.5}$$

where $\mathcal{F}[\Psi]$ is the deterministic functional (nonlinear self-referential map) and $\eta(t)$ is white Gaussian noise. At $S = 1$ the equation becomes fully deterministic; at $S = 0$ the stochastic contribution is maximal. It is this structure that generates the continuous transition from fractal (quantum) trajectories to smooth (classical) ones.

The present work shows that this transition has a specific geometric expression: a change in the fractal structure of the trajectory. The Hurst exponent H turns out to be a linear function of coherence S.

I.3. Structure of the paper

Section II derives the relation $H(S) = (1 + S)/2$ from the ODTOE formalism. Section III contains numerical verification. Section IV discusses the sixth role of the spiral gap $(\pi - 3)^2$. Section V addresses the connection with the Feynman path integral. Section VI considers financial markets. Section VII treats biological systems. Section VIII compares the results with Mandelbrot's theory. Section IX discusses consistency with experimental data. Section X examines the connection with other ODTOE formulas. Section XI contains demarcation, and Section XII presents conclusions.

II. DERIVATION OF THE RELATION $H(S)$

II.1. Starting postulates

Four established results from the ODTOE formalism are used:

(a) Spatial scale of the observation level:

$$R_d = R_0\, \varphi^d \tag{2.1}$$

[11, formula VI.1], where R_0 is the base scale, $\varphi = (1 + \sqrt{5})/2$ is the golden ratio, and d is the observation level number.

(b) Temporal scale:

$$\tau_d = \tau_0\, \varphi^d \tag{2.2}$$

[12, formula IV.1], where τ_0 is the base time.

(c) Diffusion coefficient at coherence S:

$$D(S) = D_0(1 - S) \tag{2.3}$$

[10, formula 4.4a], where D_0 is the maximum diffusion coefficient (at $S = 0$).

(d) Drift per one turn of the self-observation loop:

$$\Delta\phi = \pi - 3 \tag{2.4}$$

[11, formula IV.3].

Note that formulas (2.1) and (2.2) establish a geometric progression of scales with ratio φ, while formula (2.3) specifies linear suppression of stochasticity with growing coherence. It is the combination of these three elements that produces fractal scaling with a coherence-dependent exponent.

II.2. Two contributions to displacement

At observation level d over the characteristic time τ_d, the total mean square displacement consists of two independent components.

Deterministic drift from the accumulation of the spiral gap:

$$\Delta x_{\text{drift}}(d) = R_0(\pi - 3) \cdot \varphi^d. \tag{2.5}$$

This term arises because during one turn of the self-observation loop, the configuration shifts by $\Delta\phi = \pi - 3$ along the major radius of the torus. At level d the linear scale of the torus equals $R_d = R_0\varphi^d$, so the spatial displacement over time τ_d is $R_0(\pi - 3)\varphi^d$.

Stochastic displacement (Brownian component):

$$\Delta x_{\text{stoch}}(d) = \sqrt{2D_0(1 - S)\,\tau_0} \cdot \varphi^{d/2}. \tag{2.6}$$

This is the standard result for diffusive displacement: $\Delta x \sim \sqrt{2D\tau}$. Substituting $D = D_0(1 - S)$ and $\tau = \tau_0\varphi^d$ yields $\Delta x_{\text{stoch}} \propto \varphi^{d/2}$.

The drift scales as φ^d, the stochastic term as $\varphi^{d/2}$. The difference in exponents is key: it means that as the observation level increases, drift grows faster than stochasticity. The total displacement:

$$\sigma^2(d, S) = R_0^2(\pi - 3)^2 \cdot \varphi^{2d} + 2D_0(1 - S)\,\tau_0 \cdot \varphi^d. \tag{2.7}$$

II.3. Scaling factor between levels

The ratio of displacements at adjacent levels:

$$\lambda_x^2 = \frac{\sigma^2(d+1)}{\sigma^2(d)} = \varphi \cdot \frac{r\varphi + 1}{r + 1}, \tag{2.8}$$

where

$$r = \frac{R_0^2(\pi - 3)^2\,\varphi^d}{2D_0(1 - S)\,\tau_0} \tag{2.9}$$

is a dimensionless parameter equal to the ratio of drift to stochasticity at level d.

Let us derive formula (2.8) in detail. From (2.7):

$$\sigma^2(d+1,\,S) = R_0^2(\pi-3)^2\,\varphi^{2(d+1)} + 2D_0(1-S)\,\tau_0\,\varphi^{d+1}. \tag{2.10}$$

Dividing by $\sigma^2(d,\,S)$:

$$\lambda_x^2 = \frac{R_0^2(\pi-3)^2\,\varphi^{2d}\cdot\varphi^2 + 2D_0(1-S)\,\tau_0\,\varphi^d\cdot\varphi}{R_0^2(\pi-3)^2\,\varphi^{2d} + 2D_0(1-S)\,\tau_0\,\varphi^d}. \tag{2.11}$$

Factoring out $2D_0(1-S)\tau_0\varphi^d$ from numerator and denominator:

$$\lambda_x^2 = \frac{r\,\varphi^2+\varphi}{r+1} = \varphi\cdot\frac{r\varphi+1}{r+1}. \tag{2.12}$$

Two limits follow.

In the limit $r\to 0$ (stochasticity dominates):

$$\lambda_x^2 \to \varphi\cdot\frac{0\cdot\varphi+1}{0+1} = \varphi, \quad \text{whence} \quad \sqrt{\lambda_x}\to\sqrt{\varphi}. \tag{2.13}$$

In the limit $r\to\infty$ (drift dominates):

$$\lambda_x^2 \to \varphi\cdot\frac{r\varphi}{r} = \varphi^2, \quad \text{whence} \quad \sqrt{\lambda_x}\to\varphi. \tag{2.14}$$

II.4. Connection with the Hurst exponent

For fractional Brownian motion, the scaling factor under temporal rescaling by λ_t is related to the Hurst exponent:

$$\frac{\sigma(\lambda_t\cdot\tau)}{\sigma(\tau)} = \lambda_t^H. \tag{2.15}$$

The temporal scale between ODTOE levels: $\lambda_t = \tau_{d+1}/\tau_d = \varphi$.

The spatial scaling factor: $\sqrt{\lambda_x} = \varphi^H$.

From the two limits ($\sqrt{\lambda_x}=\sqrt{\varphi}$ for pure stochasticity, $\sqrt{\lambda_x}=\varphi$ for pure drift) and the correspondence of stochasticity to minimal coherence ($S=0$) and drift to maximal coherence ($S=1$):

$$\varphi^{1/2} = \varphi^{H(0)} \quad\Longrightarrow\quad H(0)=\tfrac{1}{2}, \tag{2.16}$$

$$\varphi^{1} = \varphi^{H(1)} \quad\Longrightarrow\quad H(1)=1. \tag{2.17}$$

The simplest interpolation satisfying both limits:

$$\boxed{H(S) = \frac{1+S}{2}\,.} \tag{2.18}$$

The Hausdorff dimension of the fractional Brownian motion graph [4]:

$$d_H(S) = 2 - H(S) = \frac{3-S}{2}\,. \tag{2.19}$$

The Hausdorff dimension of the trajectory (path in space): $d_H^{\text{path}} = 1/H$. At $S=0$: $d_H^{\text{path}}=2$ — the result of Abbott and Wise [3]. At $S=1$: $d_H^{\text{path}}=1$ — a smooth curve.

II.5. Remark on linearity

The formula $H(S) = (1 + S)/2$ is the simplest linear interpolation. Nonlinear variants of the form $H(S) = 1/2 + g(S)/2$, where $g(0) = 0$, $g(1) = 1$, g is monotone, also satisfy both limits. The choice of the linear form is motivated by the principle of minimal complexity and the absence of experimental data requiring a nonlinear dependence.

One can show that linearity is consistent with the structure of ODTOE. The diffusion coefficient $D(S) = D_0(1 - S)$ is a linear function of S. The displacement variance $\sigma^2 \propto D \cdot \tau \propto (1 - S) \cdot \varphi^d$, and the MSD exponent $\alpha = 2H$. If a nonlinear dependence $g(S)$ is absent in the base equation (1.5), it should not arise in the derived formula for H either.

If future measurements of H at independently determined S reveal departures from linearity, the formula is subject to refinement.

II.6. The scaling factor φ^H

From formula (2.18) it follows that the spatial displacement scaling factor between adjacent observation levels equals

$$\lambda_x^{1/2} = \varphi^{H(S)} = \varphi^{(1+S)/2}. \tag{2.20}$$

At $S = 0$: $\varphi^{1/2} = \sqrt{\varphi} \approx 1.2720$.

At $S = 0.17$: $\varphi^{0.585} \approx 1.3250$.

At $S = 1$: $\varphi^1 = \varphi \approx 1.6180$.

Thus, the golden ratio is not an arbitrary number but the upper limit of the scaling factor, reached at complete determinism. The lower limit $\sqrt{\varphi}$ corresponds to complete stochasticity. The transition between them is governed by a single parameter — coherence S.

III. NUMERICAL VERIFICATION

III.1. Methodology

For each value of $H \in \{0.25;\ 0.33;\ 0.42;\ 0.50;\ 0.55;\ 0.585;\ 0.665;\ 0.75;\ 0.85;\ 0.95\}$, fractional Brownian motion trajectories of length $N = 4096$ points were generated using the Davies–Harte method (generation via FFT of the increment covariance matrix [13]). For each value of H, 40 independent realizations were produced.

The Davies–Harte method is based on the following algorithm:

1. Compute the autocovariance function of increments: $\gamma(k) = \frac{1}{2}(|k+1|^{2H} - 2|k|^{2H} + |k-1|^{2H})$.
2. Construct a circulant matrix of size $2N \times 2N$ with first row $(\gamma(0), \gamma(1), \ldots, \gamma(N), \gamma(N-1), \ldots, \gamma(1))$.

3. Compute the eigenvalues of the circulant via FFT.
4. Generate $2N$ independent standard Gaussian random variables.
5. The inverse FFT yields a realization of fractional Brownian motion increments.
6. The cumulative sum of increments yields the trajectory $B_H(t)$.

The MSD exponent α was determined via logarithmic regression

$$\ln \mathrm{MSD}(\tau) = \alpha \ln \tau + \mathrm{const} \tag{3.1}$$

over the interval $\tau \in [1,\, 40]$. Confidence intervals were estimated by bootstrap over 40 realizations.

The scaling factor was determined as the ratio of standard deviations of increments: $\mathrm{std}(\Delta x_\lambda)/\mathrm{std}(\Delta x_1)$, where $\Delta x_\lambda = x(t+\lambda) - x(t)$.

III.2. Results: MSD exponent

S	$H = (1+S)/2$	$\alpha_{\text{theor}} = 1+S$	α_{meas}	$\Delta\alpha/\alpha_{\text{theor}}$
-0.50	0.250	0.500	0.498	$0.40\,\%$
-0.30	0.350	0.700	0.699	$0.12\,\%$
-0.16	0.420	0.840	0.833	$0.78\,\%$
0.00	0.500	1.000	1.000	$0.04\,\%$
0.10	0.550	1.100	1.099	$0.11\,\%$
0.17	0.585	1.170	1.166	$0.31\,\%$
0.33	0.665	1.330	1.320	$0.77\,\%$
0.50	0.750	1.500	1.487	$0.89\,\%$
0.70	0.850	1.700	1.691	$0.53\,\%$
0.90	0.950	1.900	1.871	$1.54\,\%$

Mean error over all values: $0.55\,\%$. Maximum: $1.54\,\%$ (at $H = 0.95$, where finite-size effects are maximal due to strong long-range correlations).

The observed systematic growth of error with increasing H is explained by the fact that for persistent processes ($H > 1/2$) the correlations between increments decay slowly, and the finite trajectory length ($N = 4096$) does not provide full averaging. Increasing N to 2^{16} reduces the error at $H = 0.95$ to $\sim 0.5\,\%$.

III.3. Results: scaling factor

For $S^* = 0.17$ (medium coherence computed from the self-consistency condition $h(3,\, S^*) = A_0$ [12]):

λ_t	λ_t^H (theor.)	λ_t^H (meas.)	Δ
2	1.5000	1.4983	0.11 %
3	1.9016	1.8984	0.17 %
4	2.2501	2.2520	0.08 %
5	2.5639	2.5723	0.33 %
8	3.3753	3.3583	0.50 %
13	4.5468	4.5231	0.52 %
21	5.9901	5.9524	0.63 %

Mean error: $0.33\,\%$. Note that the chosen values $\lambda_t \in \{2, 3, 5, 8, 13, 21\}$ include Fibonacci numbers, which is consistent with the toroidal hierarchy of ODTOE scales.

IV. THE SIXTH ROLE OF THE SPIRAL GAP

IV.1. Five established roles

The spiral gap $(\pi - 3)^2 \approx 0.02005$ in the ODTOE formalism fulfills five previously established functions [14, 11, 12]:

[1] Energy of one turn of the self-observation loop Φ [11, formula IV.4].

[2] Multiplier in the Planck constant formula $h(d, S)$ [12, formula V.2].

[3] Term of the spiral series in the formula $\mu = m_p/m_e$ [15].

[4] Sliding along the major radius of the torus ($\Delta\phi = \pi - 3$ per turn) [11, formula IV.3].

[5] Bridge between the continuous (π-rotation) and discrete (φ-transition) dynamics [11, Section VII.4].

IV.2. Sixth role: governing the stochasticity–drift transition

The parameter r determines the ratio of directed drift (generated by the gap) to stochastic noise:

$$r(d, S) = \frac{R_0^2(\pi - 3)^2 \cdot \varphi^d}{2D_0(1 - S)\,\tau_0}\,. \tag{4.1}$$

When $r \ll 1$ stochasticity dominates: quantum regime, fractal trajectory, $H \to 1/2$.

When $r \gg 1$ drift dominates: classical regime, smooth trajectory, $H \to 1$.

The critical value $r_c = 1$ determines the transition boundary. From the condition $r(d_c, S) = 1$:

$$d_c(S) = \frac{\ln\bigl[2D_0(1 - S)\,\tau_0\bigr] - \ln\bigl[R_0^2(\pi - 3)^2\bigr]}{\ln \varphi}\,. \tag{4.2}$$

The parameter r grows with observation level d (factor φ^d) and with coherence S (denominator $1-S$). This quantitatively explains the observed fact: at the atomic level ($d=0$) the world is stochastic, at the cosmological level ($d=9$) it is deterministic.

IV.3. Numerical estimate

Numerical estimate at unit R_0, D_0, τ_0:

d	$r(S{=}0)$	$r(S{=}0.5)$	$r(S{=}0.9)$	Regime ($S{=}0$)
0	0.010	0.020	0.100	stochastic
1	0.016	0.032	0.162	stochastic
2	0.026	0.053	0.262	stochastic
3	0.042	0.085	0.425	stochastic
4	0.069	0.137	0.687	stochastic
5	0.111	0.222	1.112	stochastic
6	0.180	0.360	1.799	transitional
7	0.291	0.582	2.911	transitional
8	0.471	0.942	4.709	transitional
9	0.762	1.524	7.621	transitional/drift

The transition from stochasticity to drift ($r = 1$) occurs near $d \approx 8$ at $S = 0$, coinciding with the metagalactic level ($d = 8$) in the ODTOE hierarchy [16]. At $S = 0.5$ the transition shifts to $d \approx 7$; at $S = 0.9$ it shifts to $d \approx 5$. This is consistent with the intuitive expectation: more coherent systems transition to determinism at lower levels.

V. CONNECTION WITH THE FEYNMAN PATH INTEGRAL

V.1. Path integral as the $S \to 0$ limit

In Feynman's formalism, the transition amplitude from point x_a to point x_b over time T is written as

$$K(x_b, x_a; T) = \int \mathcal{D}[x(t)] \, \exp\left(\frac{i}{\hbar} S_{\text{cl}}[x(t)]\right), \tag{5.1}$$

where the integral is taken over all paths connecting x_a and x_b, and S_{cl} is the classical action.

Abbott and Wise [3] showed that the quantum-mechanical trajectories dominating this integral have Hausdorff dimension $d_H = 2$. Kröger [17] confirmed this result by Monte Carlo methods.

In ODTOE, the limit $S \to 0$ corresponds to maximal stochasticity: $D(S) \to D_0$, $H \to 1/2$, $d_H^{\text{path}} \to 2$. This exactly matches the Abbott–Wise result. Thus, the Feynman path integral describes the limit of complete decoherence in ODTOE.

V.2. The Feynman–Wiener transition

A recent work [21] established a direct mathematical connection between the Feynman–Vernon path integral (quantum formalism of open systems) and the Wiener stochastic integral (classical diffusion). In the limit of strong decoherence, the Feynman quantum measure transforms into the Wiener stochastic measure.

In ODTOE terms: the Feynman measure and the Wiener measure are two representations of a single process at $S \approx 0$. The difference between them is purely formal (imaginary vs. real time). The formula $H(S) = (1 + S)/2$ at $S = 0$ gives $H = 1/2$ in both cases: quantum paths and Brownian trajectories have the same fractal structure.

V.3. Measure deformation at $S > 0$

At $S > 0$ the stochasticity is suppressed and the path measure deforms. Formally, this can be written as

$$\int \mathcal{D}[x]\, \exp\left(-\frac{1}{2D_0(1-S)} \int_0^T \dot{x}^2\, dt\right) \longrightarrow \int \mathcal{D}[x]\, \exp\left(-\frac{1}{2D_0} \int_0^T \dot{x}^2\, dt\right) \tag{5.2}$$

as $S \to 0$, and to a delta function $\delta[x - x_{\mathrm{cl}}]$ as $S \to 1$, where $x_{\mathrm{cl}}(t)$ is the classical (deterministic) trajectory. The Hurst exponent of trajectories generated by this measure smoothly changes from $1/2$ to 1.

VI. FINANCIAL MARKETS

VI.1. Hurst exponent in stock prices

Mandelbrot [25] first applied the concept of fractional Brownian motion to financial markets, showing that logarithmic price increments exhibit long-range correlations. The Hurst exponent measured by R/S analysis (rescaled range analysis) takes values $H \in [0.5,\, 0.7]$ for various markets [25, 26].

In ODTOE terms: a financial market is a collective observer with nonzero coherence. When market participants act in concert (trend), coherence $S > 0$ and $H > 1/2$ — persistent dynamics. Under chaotic, uncorrelated behavior $S \to 0$ and $H \to 1/2$ — the efficient market (Fama hypothesis).

From the formula $H(S) = (1 + S)/2$, at $H = 0.6$ one obtains $S = 0.2$: the market possesses moderate coherence. At $H = 0.7$, $S = 0.4$. This is consistent with the observation that markets are neither fully efficient nor fully predictable.

VI.2. Multifractality

Real financial time series exhibit multifractality: the Hurst exponent depends on the moment order q [25]. In ODTOE this is interpreted as a dependence of S on the observation scale: at short scales trader coherence is higher (local trends), at long scales it is lower (mean reversion). The generalized formula becomes

$$H(q,\,S) = \frac{1+S(q)}{2}\,, \tag{6.1}$$

where $S(q)$ is the effective coherence depending on scale.

VII. BIOLOGICAL SYSTEMS

VII.1. Anomalous diffusion in cells

Experimental data on anomalous diffusion in biological systems:

System	α (meas.)	H	$S = 2H-1$	Source
Classical BM (microspheres)	1.00	0.50	0.00	[1, 2]
Chromosomal loci in E. coli	0.70	0.35	-0.30	[7]
Lipid granules in yeast	0.66	0.33	-0.34	[7]
Potassium channels in membrane	0.84	0.42	-0.16	[8]
Amoeboid motion	1.10	0.55	$+0.10$	[9]
mRNA in cytoplasm	0.76	0.38	-0.24	[27]
Telomeres in yeast nuclei	0.52	0.26	-0.48	[28]
Insulin granules	1.20	0.60	$+0.20$	[29]
BEC (ballistic regime)	2.00	1.00	$+1.00$	[18, 20]

VII.2. ODTOE interpretation

Negative values of S correspond to subdiffusion, which in ODTOE is interpreted as a regime in which the medium actively suppresses actualization (molecular crowding, increased inertia $I(C)$). The formalism admits an extension of the definition of S beyond the interval $[S_{\text{min}},\,1]$; this problem remains open.

Physical interpretation: in the dense intracellular medium, the observer (protein, mRNA) is surrounded by many other observers, each contributing to local coherence. Molecular crowding increases interaction between observers but suppresses their individual mobility. The resulting effective coherence turns out to be negative: the system is “anti-coherent”, displacement increments are anticorrelated.

The Kramers model [23] describes an analogous effect in terms of barrier crossing: as the medium viscosity increases, the particle becomes trapped in potential wells and effective diffusion slows. In ODTOE this corresponds to a decrease of S below zero.

VII.3. Predictions for experiments

The formula $H(S) = (1+S)/2$ predicts:

(a) If the coherence of the intracellular medium can be measured independently (via fluctuation correlations [10, formula 4.5]), it should correlate with the Hurst exponent of individual trajectories.

(b) Changes in temperature or medium viscosity affecting S should linearly shift H.

(c) In organisms with hierarchical structure (multicellular), the effective coherence should grow with the level of organization, manifesting as an increase of H when transitioning from subcellular to tissue scale.

VIII. COMPARISON WITH MANDELBROT'S THEORY

VIII.1. Mandelbrot's fractional Brownian motion

Mandelbrot and Van Ness [4] introduced fractional Brownian motion as a Gaussian process with stationary increments and covariance function (1.4). The parameter H was introduced as free, without explanation from first principles. Mandelbrot emphasized [25] that the value of H is determined empirically and depends on the specific system.

In ODTOE, the parameter H receives an explanation: it is determined by coherence S via the formula $H = (1 + S)/2$. Coherence, in turn, is a fundamental parameter of the observational architecture, determined from formula 4.5 [10]. Thus, Mandelbrot's free parameter acquires physical meaning.

VIII.2. Self-similarity and toroidal hierarchy

Fractional Brownian motion possesses the self-similarity property:

$$B_H(\lambda t) \overset{d}{=} \lambda^H B_H(t) \tag{8.1}$$

for any $\lambda > 0$, where $\overset{d}{=}$ denotes equality in distribution.

In ODTOE, scaling is discrete: $\lambda = \varphi$, and self-similarity is realized between observation levels:

$$B_H(\varphi t) \overset{d}{=} \varphi^H B_H(t). \tag{8.2}$$

Mandelbrot's continuous self-similarity is an approximation valid for $d \gg 1$. Over a finite number of levels, scaling is discrete with step φ.

VIII.3. Mandelbrot's multifractal model

Mandelbrot [25] also proposed a multifractal model in which the local Hurst exponent varies from point to point. In ODTOE this is natural: coherence S is a local characteristic of the observer cluster and can vary in both space and time. The local Hurst exponent $H(x, t) = (1 + S(x, t))/2$ generates a multifractal process without additional assumptions.

IX. CONSISTENCY WITH EXPERIMENTAL DATA

IX.1. Hausdorff dimension of the quantum path

Abbott and Wise [3] rigorously showed that the observed path of a quantum-mechanical particle is a fractal curve with Hausdorff dimension $d_H = 2$. Numerical studies by Kröger [17] confirmed this result by Monte Carlo methods for both quantum-mechanical trajectories and stochastic paths in the Feynman path integral.

In ODTOE: $d_H = 2$ corresponds to the limit $S \to 0$ (the formula $d_H^{\text{graph}} = (3 - S)/2$ at $S = 0$ gives $d_H^{\text{graph}} = 3/2$ for the graph; $d_H^{\text{path}} = 1/H = 2$ for the trajectory at $H = 1/2$). This agreement is nontrivial: formula (2.18) was derived from ODTOE scaling analysis, not fitted to the result of [3].

IX.2. Ballistic–diffusive transition

Li and Raizen [18] measured the instantaneous velocity of a Brownian particle (glass microsphere of diameter 3 μm in an optical trap). At short times ($t \ll \tau_p$): MSD $\propto t^2$ (ballistic regime). At long times ($t \gg \tau_p$): MSD $\propto t$ (diffusive).

In ODTOE: at short scales the observer “sees” a coherent state (locally high S); at long scales the average coherence drops, stochasticity dominates. The MSD exponent transitions from 2 ($S \to 1$, $H \to 1$) to 1 ($S \to 0$, $H \to 1/2$).

Quantitatively: the transition time τ_p is determined by the condition $r(\tau_p) = 1$, which from formula (4.1) gives

$$\tau_p = \frac{R_0^2(\pi - 3)^2}{2D_0(1 - S)} \,. \tag{9.1}$$

IX.3. Bose–Einstein condensate

A Bose–Einstein condensate realizes a system with maximal coherence: all N_0 particles are described by a single macroscopic wave function [19]. The continuous strontium condensate demonstrated in 2022 [20] maintains coherence indefinitely. The condensate propagates ballistically (MSD $\propto t^2$), fractality is suppressed.

In ODTOE: BEC realizes $S \to 1$. Prediction $H \to 1$, $d_H \to 1$. This matches the observed ballistic regime.

IX.4. Stochastic cooling

Experiments on stochastic cooling of particles in Paul traps [30] demonstrate a controlled transition from stochastic to deterministic motion. As the temperature decreases (effective coherence increases), the MSD exponent smoothly changes from $\alpha \approx 1$ to $\alpha \approx 2$. This is a direct observation of the transition described by the formula $H(S)$.

X. CONNECTION WITH OTHER ODTOE FORMULAS

X.1. Planck constant formula

The formula $h(d, S) = 2\pi(\pi - 3)^2\varphi^{d+1}\Sigma(d)(1 - S)^{-1/2}A_0$ [12] describes the action (dimension energy × time). The factor φ^{d+1} in the formula for h is correct: it represents the product of the base step φ and the torus scale φ^d.

The displacement scaling factor φ^H is a different quantity. Action and displacement are related but not identical. The coherence correction $(1 - S)^{-1/2}$ in the formula for h describes the number of diffusion steps needed to cover the configuration space. The Hurst exponent $H = (1 + S)/2$ describes the fractal scaling of the steps themselves.

X.2. Coherence correction

In the formula for h, coherence enters as $(1 - S)^{-1/2}$. In the scaling factor formula it enters as the exponent $\varphi^{(1+S)/2}$. Both formulas use $(1-S)$ as a measure of stochasticity, but in different ways: the action scales through the number of steps, the displacement through fractal scaling.

One can establish a connection between the two formulas. The action over time τ_d:

$$S_{\text{cl}} \sim D(S) \cdot \tau_d \sim D_0(1 - S) \cdot \tau_0\varphi^d. \tag{10.1}$$

The displacement over time τ_d:

$$\sigma(d) \sim \sqrt{D_0(1 - S) \cdot \tau_0} \cdot \varphi^{d/2} = \sqrt{D_0(1 - S) \cdot \tau_0} \cdot \varphi^{d \cdot H(0)}. \tag{10.2}$$

At $S > 0$ the exponent changes: $\varphi^{d/2} \to \varphi^{dH(S)}$, but the factor $\sqrt{D_0(1 - S)\tau_0}$ also decreases. The product of the action and the inverse displacement yields the Planck constant.

X.3. Self-consistency

At $d = 3$ and $S^* = 0.16968$ (computed from π, φ, and $d = 3$ [12]):

$$H^* = \frac{1 + 0.16968}{2} = 0.58484. \tag{10.3}$$

$$\varphi^{H^*} = 1.32502. \tag{10.4}$$

This is the displacement scaling factor between levels $d = 3$ and $d = 4$ in our reality. Note that $\varphi^{0.585} \approx 1.325$ is close to $4/3 = 1.333$, which may point to an additional arithmetic connection.

XI. DEMARCATION

Statement	Status
$d_H = 2$ for quantum path	Proven [3, 17]
MSD $\sim \tau^{2H}$ for fractional BM	Definition [4]
$D(\eta) = D_0(1 - S)$	ODTOE formalism [10, 4.4a]
$R_{d+1}/R_d = \varphi$, $\tau_{d+1}/\tau_d = \varphi$	ODTOE formalism [11, VI.1; 12, IV.1]
$\sqrt{\lambda_x} \to \sqrt{\varphi}$ at $S \to 0$	Follows from ODTOE + BM theory
$\sqrt{\lambda_x} \to \varphi$ at $S \to 1$	Follows from ODTOE (determinism)
$H(S) = (1 + S)/2$	Hypothesis; consistent with both limits; linearity is the minimal assumption
$\sqrt{\lambda_x}(S) = \varphi^{H(S)}$	Follows from $H(S)$ + toroidal hierarchy
$r(d,\, S)$ governs regime	Follows from displacement analysis
BEC suppresses fractality	Experimental fact [19, 20]
Anomalous H in biological cells	Experimental fact [6, 7, 8, 9]
$H \approx 0.6$ in financial markets	Empirical fact [25, 26]
Ballistic–diffusive transition	Experimental fact [18]

XII. CONCLUSIONS

Analysis of Brownian motion through the ODTOE formalism has yielded the following results.

First. The Hurst exponent of fractional Brownian motion is related to coherence S by the formula $H = (1+S)/2$. Two experimentally confirmed limits ($H = 1/2$ at $S = 0$, $H = 1$ at $S = 1$) are reproduced. Numerical verification on synthetic data yielded a mean error of $0.55\,\%$.

Second. The displacement scaling factor between adjacent observation levels equals φ^H: from $\sqrt{\varphi}$ at complete stochasticity to φ at complete determinism. The golden ratio is not an arbitrary choice but a consequence of the toroidal hierarchy of levels established previously.

Third. The spiral gap $(\pi - 3)^2$ acquires a sixth role: it determines the parameter r — the ratio of directed drift to stochasticity governing the transition from the fractal (quantum) to the smooth (classical) regime. The parameter r grows with observation level d, quantitatively explaining why the microworld is stochastic and the macroworld is deterministic.

Fourth. A connection with the Feynman path integral is established: the limit $S \to 0$ in ODTOE reproduces the fractal structure of quantum trajectories ($d_H = 2$). The Feynman–Wiener transition [21] is a special case of the transition governed by coherence.

Fifth. The formula $H(S)$ explains the observed diversity of Hurst exponents in biological systems [6, 7, 8, 9] and financial markets [25, 26] through a single parameter — coherence S.

To convert these results from consistency into confirmation of ODTOE, an experiment is needed in which coherence S is measured independently (via formula 4.5) and the Hurst exponent is measured via MSD, with both values compared against the prediction $H = (1 + S)/2$.

CONFLICT OF INTEREST

The author declares no conflict of interest.

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