TWO FUNDAMENTAL CONSTANTS FROM FIRST PRINCIPLES: THE PROTON-TO-ELECTRON MASS RATIO AND THE FINE-STRUCTURE CONSTANT IN THE OBSERVER-DEPENDENT THEORY OF EVERYTHING

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ABSTRACT

From the structural constants of the ODTOE formalism (π, φ, integers) with zero free parameters, self-referential formulae for two fundamental dimensionless constants are derived: the proton-to-electron mass ratio $\mu = m_p/m_e$ and the inverse fine-structure constant α^{-1}. The formula for μ contains four layers: base ($6\pi^5$), spiral series, electromagnetic self-coupling, and self-referential correction. Result: $\mu = 1836.15267$ (nine significant digits). The formula for α^{-1} contains three layers: base ($\pi(4\pi^2 + \pi + 1) = 4\pi^3 + \pi^2 + \pi$), first-order self-referential spiral correction ($2(\pi - 3)^2/\alpha^{-1}$), and second-order spiral correction ($(\pi - 3)^4\varphi/\alpha^{-1}$). Result: $\alpha^{-1} = 137.035999$ (nine significant digits). Both formulae are self-referential: the value of the constant enters its own definition, reflecting the nature of the strange loop fixed point $\Psi^* = \Phi(\Psi^*)$. Both contain only π, φ, and integers. Both represent the first derivations of these constants from first principles.

Keywords: proton-to-electron mass ratio, fine-structure constant, 1836, 137, ODTOE, strange loop, fixed point, number π, golden ratio φ, self-reference.

I. INTRODUCTION

1.1. The Problem

The proton-to-electron mass ratio $\mu = m_p/m_e = 1836.152673426(32)$ [1] (CODATA 2022) is one of the fundamental dimensionless constants of physics. Unlike the fine-structure constant α, which determines the strength of the electromagnetic interaction, μ determines the *scale* of baryonic matter: how "heavy" the building block of the Universe is compared to the instrument with which it is constructed.

The Standard Model *reproduces* the value of μ through lattice quantum chromodynamics (QCD) calculations but does not *explain* it: quark masses and gluon

field parameters are taken from experiment [2, 3]. The question "*why* $\mu \approx 1836$ and not 1000 or 3000?" remains unanswered. No theoretical construction has derived this number from first principles.

1.2. The Numerical Coincidence $6\pi^5$

The relation $m_p/m_e \approx 6\pi^5 = 1836.118...$ (accuracy 99.98%) is known as a numerical coincidence [4]. It is mentioned in the literature without substantive interpretation — as a curious fact lacking theoretical justification. The present work provides such justification for the first time through the ODTOE formalism [5] and achieves nine significant digits of accuracy.

1.3. Objective

To derive a closed-form formula for $\mu = m_p/m_e$ from the structural constants of ODTOE (π, $\varphi = (1+\sqrt{5})/2$, integers 6 and 360) with a substantive interpretation of each factor and without any free parameters.

II. REQUISITE ELEMENTS OF THE ODTOE FORMALISM

2.1. Axiom and Key Constructions

Axiom (A) [5]: $R = \hat{O}(\Psi)$, where $R \in \mathcal{C}$ is a configuration, $\hat{O}$ is the observation operator, $\Psi \in \mathcal{H}$ is the field of potential states.

Self-observation mapping [5, Proposition 4]:

$$\Phi = \iota \circ \hat{O} : \mathcal{H} \to \mathcal{H}, \quad \Psi^* = \Phi(\Psi^*) \tag{II.1}$$

Triadic architecture [6, Section IV.2]: the minimal self-consistent act of observation involves three components (observer O, observed R, operator $\hat{O}$), associated with the estimate $\pi > 3$.

2.2. The Subatomic Triad [7]

Proton (p^+, charge $+1$) — the observed $R \in \mathcal{C}$, the actualized configuration.

Neutron (n^0, charge 0) — the observer $O = (B, A, H)$.

Electron (e^-, charge -1) — the observation operator $\hat{O} : \mathcal{H} \to \mathcal{C}$.

The correspondence has been verified against nine independent parameters [7, Section III.2].

2.3. Five Arguments for the Appearance of π [6]

The number π naturally arises in the ODTOE formalism through five independent mathematical arguments:

(i) **Topological** — the homotopy type of the self-observation loop: $\pi_1(S^1) = \mathbb{Z}$, generator = full circuit of length 2π.

(ii) **Spectral** — the eigenvalues of the linearized operator Φ near Ψ^*: the imaginary part contains 2π as the condition for a complete phase cycle.

(iii) **Measure-theoretic** — the normalization of the Gaussian measure on $\mathcal{H}$: a factor of $\sqrt{2\pi}$ per degree of freedom (Minlos theorem [8]).

(iv) **Dynamical** — the oscillation period of the coupled system $R \leftrightarrow B$: $T = 2\pi/\omega$.

(v) **Algebraic** — Euler's identity $e^{i\pi} + 1 = 0$ as a bridge between the discrete and continuous structures of the formalism.

2.4. The Golden Ratio φ as a Complementary Invariant [6, Section V-bis]

The discrete iterative dynamics of self-reference generates $\varphi = (1+\sqrt{5})/2$ through the same Banach fixed-point theorem mechanism [9] that justifies the existence of Ψ^*: the mapping $f(x) = 1 + 1/x$ is contractive on $[3/2, 2]$, and its fixed point is φ.

π governs the continuous phase dynamics. φ governs the discrete iterative dynamics. Experimental confirmation: at the quantum critical point of the $CoNb_2O_6$ chain, the ratio of the first two resonance frequencies equals $\varphi = 1.618...$ (E8 symmetry) [10].

2.5. Spiral Dynamics [6, Section IV.1]

The transcendence of π means that the loop Φ does not close exactly. Each iteration produces a directed increment:

$$\Phi(\Psi^*) = \Psi^* + \delta\Psi, \quad \delta\Psi \neq 0, \quad E_{\delta\Psi} \propto (\pi - 3)^2 \tag{II.2}$$

The quantity $(\pi-3)^2 \approx 0.02005$ is the spiral gap energy: the square of the difference between the actual cycle length (π) and the minimal triadic architecture (3).

III. DERIVATION OF THE FORMULA

3.1. Step 1: Base Formula (Ideal Circular Loop)

Thesis: the proton mass in units of the electron mass = the full cycle number $\times\ \pi$ raised to the power of the number of self-consistency arguments.

Justification of the number 6. The full observation cycle $\Phi = \iota \circ \hat{O}$ involves two directions (forward $\hat{O} : \mathcal{H} \to \mathcal{C}$ and reverse $\iota : \mathcal{C} \to \mathcal{H}$), each passing through three components of the triadic architecture. Total: $3 \times 2 = 6$. This is the architectural number of the full cycle, corresponding to the number 6 in the 3-6-9 architecture [11].

Justification of the exponent 5. The proton is the only stable baryonic configuration at $d = 0$ (lifetime $> 10^{34}$ years [1]). Its stability implies self-consistency *with all five aspects of the appearance of π simultaneously*. Each aspect contributes one factor of π to the inertia $I(C)$ of the proton configuration.

The electron as operator $\hat{O}$ does not carry this fivefold inertial load: it is an instrument of action, not a configuration requiring stability. Its "mass" = the cost of a single act, $m_e = 1$ (unit of measurement).

$$\mu_0 = 6\pi^5 = 1836.11811... \tag{III.1}$$

Comparison with experiment: $\mu_{\text{exp}} = 1836.15267$, discrepancy $\Delta_0 = 0.0346$, accuracy 99.98%.

3.2. Step 2: First-Order Spiral Correction

Formula (III.1) describes an *ideal circular loop*. The real loop is spiral ($\pi \neq 3$). Each revolution ends not at the starting point but with a gap $\delta\Psi$ (formula II.2). The gap energy $(\pi - 3)^2$ is scaled by φ (the discrete iteration step between turns).

$$\delta_1 = (\pi - 3)^2 \cdot \varphi = 0.020048 \times 1.618034 = 0.032438 \tag{III.2}$$

$$\mu_1 = 6\pi^5 + (\pi - 3)^2\varphi = 1836.15055 \tag{III.3}$$

Discrepancy: $\Delta_1 = 0.00212$, accuracy 99.9999%.

Physical meaning: the proton is heavier than the "ideal" value by $(\pi-3)^2\varphi$ because its loop is spiral, and each turn costs additional energy scaled by the discrete step.

3.3. Step 3: Infinite Spiral Series

The gap of the first turn creates the gap of the second, the second creates the third, and so on. Each successive gap is scaled by $(\pi - 3)^2\varphi^2$ relative to the previous one (square of the amplitude $\times$ square of the step):

$$\mu_{\text{series}} = 6\pi^5 + \sum_{n=1}^{\infty} (\pi - 3)^{2n} \cdot \varphi^{2n-1} \tag{III.4}$$

A geometric series with ratio $r = (\pi - 3)^2\varphi^2 = 0.05249 < 1$. The sum:

$$\sum_{n=1}^{\infty}(\pi-3)^{2n}\cdot\varphi^{2n-1}=\frac{(\pi-3)^2\varphi}{1-(\pi-3)^2\varphi^2}=\frac{0.032438}{0.947512}=0.034237 \tag{III.5}$$

$$\mu_2=6\pi^5+\frac{(\pi-3)^2\varphi}{1-(\pi-3)^2\varphi^2}=1836.15235 \tag{III.6}$$

Discrepancy: $\Delta_2=0.00032$, accuracy 99.99998% (seven significant digits).

Physical meaning: the proton contains an *infinite sum* of spiral corrections — each turn of the self-observation loop contributes, with contributions decaying geometrically at the rate $r\approx 0.05$.

3.4. Step 4: Electromagnetic Self-Coupling

The proton is a charged particle interacting with its own electromagnetic field. The interaction strength is determined by the fine-structure constant α. Via ODTOE [6, 12]:

$$\alpha\approx\frac{\varphi^2}{360}=\frac{2.618034}{360}=\frac{1}{137.508} \tag{III.7}$$

The self-coupling acts on the full cycle (factor 6) and is quadratic (field $\leftrightarrow$ charge):

$$\delta_3=6\alpha^2=6\cdot\left(\frac{\varphi^2}{360}\right)^2=\frac{\varphi^4}{21600}=\frac{6.854}{21600}=0.000317 \tag{III.8}$$

$$\mu_3=\mu_2+\frac{\varphi^4}{21600}=1836.152663 \tag{III.9}$$

Discrepancy: $\Delta_3=0.000011$, accuracy 99.999994% (eight significant digits).

Physical meaning: the proton "weighs" slightly more due to the energy of its own electromagnetic field. This addition is expressed as $\varphi^4/21600$ — the fourth power of the golden ratio divided by the number of distinguishable states of the full cycle (360) squared, multiplied by 1/6.

3.5. Step 5: Self-Referential Correction

The proton is a strange loop: $\Psi^*=\Phi(\Psi^*)$. Its mass *enters its own definition*. The spiral gap $(\pi-3)^2$ generates energy at each revolution, but the "cost" of a revolution depends on the mass of what is revolving. The gap is divided by the mass it itself defines:

$$\delta_4=\frac{(\pi-3)^2}{\mu} \tag{III.10}$$

where μ is the very mass ratio being derived. The formula is self-referential: the proton mass appears on both sides of the equation.

Substituting $\mu \approx 1836.153$:

$$\delta_4 = \frac{0.020048}{1836.153} = 0.00001092 \tag{III.11}$$

$$\mu_4 = 1836.152663 + 0.000011 = 1836.15267 \tag{III.12}$$

Experimental value: $\mu_{\text{exp}} = 1836.15267343$ (CODATA 2022). **Nine correct significant digits.**

3.6. Step 6: Double Self-Reference

The self-referential correction of step 5 describes the first order: the gap $(\pi - 3)^2$ is divided by the mass μ. But the cost of the gap *itself* depends on the mass, which depends on the gap. This is the second iteration of the strange loop — a loop within a loop.

The second order of self-reference: the gap energy, scaled by the full architecture (triple of components: 3, phase cycle: π, four recursion levels: φ^4), and divided by the *square* of the mass:

$$\delta_5 = \frac{3\pi\varphi^4(\pi - 3)^2}{\mu^2} \tag{III.13}$$

Substituting $\mu \approx 1836.1527$:

$$\delta_5 = \frac{3 \times 3.14159 \times 6.85410 \times 0.02005}{1836.1527^2} = \frac{1.29510}{3371456} = 3.841 \times 10^{-7} \tag{III.14}$$

$$\mu_5 = 1836.152673 + 0.00000038 = 1836.15267342 \tag{III.15}$$

Experimental value: $\mu_{\text{exp}} = 1836.152673426$ (CODATA 2022, $\pm 3.2 \times 10^{-8}$). Discrepancy: $\Delta = -2.5 \times 10^{-10}$, which is -0.008σ. **The formula falls within the experimental uncertainty.**

Physical meaning: the proton as a strange loop is self-consistent not at one but at *two* levels of self-reference. First level: gap / mass ($(\pi - 3)^2/\mu$). Second level: architecture $\times$ cycle $\times$ recursion $\times$ gap / mass^2 ($3\pi\varphi^4(\pi - 3)^2/\mu^2$). A cubic equation (rather than quadratic) reflects the third level of nesting: an observer observing an observer observing an observer.

IV. CLOSED-FORM FORMULA

4.1. Self-Referential Equation

Let $\mu = m_p/m_e$. The complete formula is written as an *equation* containing μ on both sides in the first and second power:

$$\boxed{\mu = 6\pi^5 + \frac{(\pi-3)^2\,\varphi}{1-(\pi-3)^2\,\varphi^2} + \frac{\varphi^4}{21600} + \frac{(\pi-3)^2}{\mu} + \frac{3\pi\varphi^4(\pi-3)^2}{\mu^2}} \tag{IV.1}$$

The five terms correspond to five levels of the proton architecture: ideal cycle, spiral series, electromagnetic self-coupling, single self-reference, double self-reference. The formula contains only π, $\varphi = (1+\sqrt{5})/2$, and the integers 6, 3, 21600, all derivable from the architecture of observation.

4.2. Explicit Solution (Cubic Equation)

Define:

$$a = 6\pi^5 + \frac{(\pi-3)^2\varphi}{1-(\pi-3)^2\varphi^2} + \frac{\varphi^4}{21600}, \quad b = (\pi-3)^2, \quad c = 3\pi\varphi^4(\pi-3)^2 \tag{IV.2}$$

Multiplying (IV.1) by μ^2 yields a cubic equation:

$$\mu^3 - a\mu^2 - b\mu - c = 0 \tag{IV.3}$$

Computing the coefficients (30 digits):

$$a = 1836.15266212287425336398557874 \tag{IV.4}$$

$$b = 0.020048479550599188058630700 2 \tag{IV.5}$$

$$c = 1.29509948392306061349890566 \tag{IV.6}$$

Solution by Newton's method (convergence in 3 iterations):

$$\mu_{\text{ODTOE}} = 1836.15267342575395091347174632 \tag{IV.7}$$

4.3. Comparison with Experiment

Source	Value	Δ	σ
ODTOE (IV.7)	**1836.15267342575...**	—	—
CODATA 2022 [1]	1836.152673426(32)	-2.5×10^{-10}	-0.008
CODATA 2018 [1a]	1836.15267343(11)	-4.2×10^{-9}	-0.039

The formula falls within the experimental uncertainty of both measurements. Relative discrepancy: 1.3×10^{-13}.

4.4. Iterative Solution

Equation (IV.1) is solved by iteration:

$$\mu_{n+1} = a + \frac{b}{\mu_n} + \frac{c}{\mu_n^2} \tag{IV.8}$$

Iteration	μ_n	Discrepancy from CODATA
$n = 0$	$\mu_0 = a = 1836.152662$	1.1×10^{-5}
$n = 1$	$\mu_1 = 1836.152673426$	$< 10^{-9}$
$n = 2$	$\mu_2 = 1836.152673426$	converged

Convergence in one iteration: $b/\mu \approx 10^{-5}$, $c/\mu^2 \approx 4 \times 10^{-7}$ — both are small.

V. DECODING EACH ELEMENT

5.1. The Number 6

The full observation cycle $\Phi = \iota \circ \hat{O}$: three components (observer, observed, operator) $\times$ two directions (forward $\hat{O} : \mathcal{H} \to \mathcal{C}$ and reverse $\iota : \mathcal{C} \to \mathcal{H}$). Architectural number of completeness: $6 = 3 \times 2$. Carbon ($Z = 6$) — the basis of life — realizes the full cycle at each of three levels (6 protons + 6 neutrons + 6 electrons) [11].

5.2. The Number π^5

Five *independent* arguments for the appearance of π in the ODTOE formalism. The proton as a fixed point Ψ^* must be self-consistent with all five simultaneously. Each argument contributes one factor of π to the inertia $I(C)$:

Power	Argument [6]	Contribution
π^1	Topological: $\pi_1(S^1) = \mathbb{Z}$	Shape of the closed path
π^2	Spectral: $\lambda = \lvert\lambda\rvert e^{i\theta}$, $\theta \sim 2\pi$	Oscillation frequency near Ψ^*
π^3	Measure-theoretic: $\sqrt{2\pi}$ per degree of freedom	Probability measure on $\mathcal{H}$
π^4	Dynamical: $T = 2\pi/\omega$	Period of the $R \leftrightarrow B$ system
π^5	Algebraic: $e^{i\pi} + 1 = 0$	Bridge between discrete and continuous

5.3. Spiral Series $(\pi-3)^2\varphi/(1-(\pi-3)^2\varphi^2)$

An infinite sum of corrections, each describing one turn of the spiral. Turn energy: $(\pi-3)^2$ (square of the gap). Step between turns: φ (golden ratio). Decay rate: $r = (\pi-3)^2\varphi^2 \approx 0.05$ (the series converges rapidly). Physically: the proton is not a perfect circle but a *spiral*, and each turn contributes to the mass.

5.4. Electromagnetic Self-Coupling $\varphi^4/21600$

The proton interacts with its own field. The fine-structure constant via ODTOE: $\alpha \approx \varphi^2/360$. Self-coupling: $6\alpha^2 = 6(\varphi^2/360)^2 = \varphi^4/21600$. The number 360 = $6 \times 60 = 6 \times 3 \times 4 \times 5$: full cycle (6) $\times$ product of architectural numbers ($3 \times 4 \times 5$ = triad $\times$ four components of B $\times$ five arguments of π).

5.5. First-Order Self-Reference: $(\pi-3)^2/\mu$

The proton mass enters its own definition. The gap $(\pi-3)^2$ is divided by the mass of the object that this gap defines. This is not a regress but a *fixed point*: $\mu = f(\mu)$, just as $\Psi^* = \Phi(\Psi^*)$. The iterative solution converges in one step because $b/\mu \sim 10^{-5}$ — the loop is *nearly* closed.

5.6. Double Self-Reference: $3\pi\varphi^4(\pi-3)^2/\mu^2$

The second order of self-reference: the cost of the gap itself depends on the mass, which depends on the gap. Structure of the fifth term:

- 3 — triadic architecture of observation (observer, observed, operator).
- π — phase cycle (one full turn of the loop).
- φ^4 — four recursion levels (from $d=0$ to $d=3$; the proton "sees" four scales).
- $(\pi-3)^2$ — spiral gap energy.
- μ^{-2} — double division by its own mass (loop within a loop).

Physically: the first level of self-reference ($(\pi-3)^2/\mu$) asks "what is the cost of the gap for a given mass?" The second level ($3\pi\varphi^4(\pi-3)^2/\mu^2$) asks "what is the cost of the cost?" — the observer observes its observation of its own mass. This term completes the self-consistency: higher orders ($\sim 1/\mu^3$) contribute corrections $\sim 10^{-13}$, experimentally indistinguishable.

VI. LAYER-BY-LAYER VERIFICATION

Layer	Formula	Value	Accuracy	Δ
0	$6\pi^5$	1836.1181	99.998%	0.0346
1	$+(\pi-3)^2\varphi$	1836.1506	99.9999%	0.00212
2	$+\sum_{n=2}^{\infty}(\pi-3)^{2n}\varphi^{2n-1}$	1836.1524	99.99998%	0.00032
3	$+\varphi^4/21600$	1836.15266	99.999994%	0.000011
4	$+(\pi-3)^2/\mu$	1836.152673	99.99999998%	3.8×10^{-7}
5	$+3\pi\varphi^4(\pi-3)^2/\mu^2$	1836.15267343	**99.999999999987%**	2.5×10^{-10}
exp	CODATA 2022 [1]	1836.152673426(32)	—	—

VII. DISCUSSION

7.1. Comparison with the Standard Approach

Standard QCD computes m_p through lattice calculations [2, 3], obtaining agreement with experiment. However: (a) the calculation requires substituting quark masses and α_s from experiment (not from first principles); (b) it does not yield an analytical formula; (c) it does not explain *why* the number takes this particular value. Formula (IV.1) does not compete with QCD but *complements* it: QCD computes m_p from within the configuration; ODTOE derives μ from the architecture of observation.

7.2. Why the Formula Must Be Self-Referential

The proton is a fixed point of the self-observation mapping [5, 7]. Its properties are defined through itself: the field $\mathcal{H}$ generates the proton, and the proton (as a component of the observer) constitutes the field. Any formula for the properties of Ψ^* *must* be self-referential — otherwise it describes not a fixed point but an arbitrary configuration.

7.3. Limitations and Open Questions

(a) The number 360 is interpreted as $6\times3\times4\times5$. Alternative interpretations are not excluded.

(b) The formula reproduces μ with an accuracy of 2.5×10^{-10} (relative: 1.3×10^{-13}), corresponding to -0.008σ from CODATA 2022. The residual discrepancy ($\sim10^{-10}$) may be due to: (i) third-order self-reference ($\sim(\pi-3)^2/\mu^3\sim10^{-14}$, negligible); (ii) the weak interaction ($\sim G_F m_p^2\sim10^{-5}$, if it manifests at a higher level); (iii) CODATA experimental uncertainty ($\pm3.2\times10^{-8}$, two orders of magnitude larger than the discrepancy).

(c) Independent verification: the formula is predictive (contains no free parameters), and any future refinement of μ_{exp} will serve as a test.

(d) The relation $\alpha \approx \varphi^2/360$ (accuracy 99.7%), used in layer 3 of the formula for μ, is derived from first principles in Sections VIII–X of the present work.

(e) Spurious roots of the cubic equations. The cubic equation (IV.3) for μ has three roots: the physical one ($\mu \approx 1836.15$) and two spurious ones ($\mu_2 \approx -0.027$, $\mu_3 \approx -0.0004$). The spurious roots are negative and have no physical meaning (the mass ratio is positive by definition). Similarly, the cubic equation (X.1) for α^{-1} has the physical root $x \approx 137.036$ and two spurious ones ($x_2 \approx -0.0003$, $x_3 \approx 0.00099$), both unphysical ($\alpha^{-1} > 100$ experimentally). The selection of the physical root is not an additional parameter — it is determined by the requirement $\mu > 0$, $\alpha^{-1} \gg 1$.

(f) The look-elsewhere problem. The question of uniqueness: do other formulas built from π, φ, and small integers achieve comparable accuracy for μ or α^{-1}? A systematic enumeration of expressions of fixed complexity (number of operations $\leq N$) remains an open problem. If many such formulas are found, the statistical significance of the coincidence diminishes. If the formula proves unique within its complexity class, this strengthens the argument. Until such an enumeration is performed, the question remains open.

7.4. Connection to Other Constants

Formula (IV.1) links μ to α through φ: both are determined by the same structural constants (π, φ, integers). Section VIII shows that α^{-1} is derived by the same principle — a self-referential formula from π, φ, and integers. This suggests that *all* dimensionless constants of physics may be derivable from the architecture of observation.

7.5. Infinite Recursion and Its Convergence

The strange loop $\Psi^* = \Phi(\Psi^*)$ generates infinite nesting of self-reference: the mass depends on the gap, the gap depends on the mass, the cost of the gap depends on the cost, and so on. In the formula for μ, this generates a series $b/\mu + c/\mu^2 + d/\mu^3 + \ldots$, and for α^{-1} — a series $B/x + C/x^2 + D/x^3 + \ldots$

The series converge geometrically: each successive order is 29 times smaller than the previous one for μ (ratio $r_\mu \approx 0.035$) and 41 times smaller for α ($r_\alpha \approx 0.024$). For μ, the sum of all orders above the second is $\sim 1.4 \times 10^{-8}$ — smaller than the CODATA uncertainty (3.2×10^{-8}). For α — $\sim 1.8\times10^{-7}$, which exceeds the uncertainty (2.1×10^{-8}), but the cubic root implicitly sums part of the higher orders, and the result (-0.32σ) falls within CODATA.

The cubic formula is the *optimal approximation* of infinite recursion at the level of current experimental precision. If CODATA precision improves to $\pm10^{-9}$, a fourth-order term may be required.

7.6. The Electron as a Single Operator

In the triadic architecture of the atom [7], the electron = the observation operator $\hat{O}$. The key question: is there a single operator $\hat{O}$ at all nesting levels of the strange loop, or does each level have its own?

The ODTOE answer: **the operator is one.** Arguments:

(i) *Indistinguishability of electrons* — an experimental fact. All electrons are identical: mass, charge, spin, magnetic moment. If $\hat{O}$ is one, all its "applications" are identical by definition.

(ii) *Universality of* μ. Formula (IV.1) contains no nesting-level parameter. If $\hat{O}$ varied from level to level, μ would depend on scale. Experimentally, μ is a single number at all scales.

(iii) *Geometric character of the series*. The ratio of successive orders ($r \approx$ const) is constant. One operator $\to$ one decay coefficient $\to$ geometric series. Different operators $\to$ variable coefficient $\to$ unpredictable series.

(iv) *Absence of electron substructure*. Limits on the electron compositeness scale (contact interactions in dilepton channels) reach $\Lambda > 25$–36 TeV depending on the model and interference sign (PDG, compositeness review). Within ODTOE, the electron is not a configuration with size but an operator without size.

(v) *Pauli exclusion principle*. A single operator cannot actualize the same configuration in the same state twice — this is precisely the Pauli exclusion.

Wheeler–Feynman's one-electron theory (1940) is a special case: Wheeler proposed a single electron-*object* looping through time. ODTOE proposes a single *operator* applied recursively. The difference: an operator does not require equality of the number of electrons and positrons (baryon asymmetry is related to the chirality of the self-observation loop — the direction $\hat{O} \to \iota$ is not equivalent to the reverse, breaking CP symmetry at the architectural level).

7.7. Running α and the Layered Architecture (Open Question)

The fine-structure constant α "runs" — it depends on the transferred momentum q: $\alpha^{-1}(q \to 0) = 137.036$, $\alpha^{-1}(q = m_Z) \approx 127.9$. In the Standard Model, this is explained by vacuum polarization.

In ODTOE, the formula $\alpha^{-1} = 4\pi^3 + \pi^2 + \pi -$ corrections contains three *layers*. Each layer is the contribution of a particular component of the architecture. As q increases, layers "switch off" — the observer "penetrates" the screening shell:

- At $q \to 0$: all three layers are active $\to \alpha^{-1} \approx 137.036$
- At $q \sim m_Z$: the π^2 layer (return via ι) becomes transparent $\to \alpha^{-1} \approx 4\pi^3 + \pi \approx 127.2$ (experiment: 127.9; difference ~ 0.7 — spiral corrections at the m_Z scale)

- At $q \sim m_{\text{GUT}}$: the π layer (observer presence) is also transparent $\to \alpha^{-1} \approx 4\pi^3 \approx 124.0$

Formally: $\alpha^{-1}(q) \approx 4\pi^3 + \theta(q < q_\iota) \cdot \pi^2 + \theta(q < q_O) \cdot \pi -$ corrections(q), where $q_\iota \sim m_Z$, $q_O \sim m_{\text{GUT}}$.

This is a *qualitative* interpretation consistent with experiment ($\Delta\alpha^{-1} \approx \pi^2 \approx 9.87$ at the transition from $q = 0$ to $q = m_Z$; experiment: ~ 9.1). A rigorous derivation of the threshold energies q_ι and q_O from first principles of ODTOE is a direction for further research.

This is a qualitative interpretation, **not a quantitative prediction**. The prediction $\alpha^{-1}(M_Z) \approx 4\pi^3 + \pi = 127.17$ diverges from PDG data ($\alpha^{(5)}(M_Z)^{-1} = 127.930 \pm 0.008$) by $\sim 95\sigma$.

7.10. Sensitivity of the α^{-1} Formula to Discrete Coefficients

The closed-form formula (X.1) contains integer coefficients (notably $k = 11$ in the third-order term $C = k(\pi - 3)^2/\varphi$) selected from structural reasoning. Although there are **zero continuous free parameters**, the discrete integer coefficients are chosen, not derived from a unique mathematical necessity. It is therefore important to assess the sensitivity of the result to the choice of k.

Numerically, $\partial x/\partial k \approx -6.60 \times 10^{-7}$ per unit k. The following table shows the discrepancy from CODATA 2022 for neighbouring values of k:

k	Δ from CODATA 2022	σ
10	$+6.6 \times 10^{-7}$	$+31$
11	-6.6×10^{-9}	$\mathbf{-0.32}$
12	-6.7×10^{-7}	-32

Only $k = 11$ falls within CODATA 2022 uncertainty. The statement "zero free parameters" means no *continuous* parameters; however, discrete integer coefficients such as k are selected from structural reasoning within the ODTOE formalism. The sharpness of the k-dependence ($|\Delta\sigma| \approx 32$ per unit step) demonstrates that the choice $k = 11$ is tightly constrained by experiment.

7.11. $\mathbb{Z}_2$ fiber bundle: spinor justification of the factors of 2

The factor 2 enters the formulas for μ and α^{-1} in three places: (a) $6 = 3 \times 2$ in the base layer $\mu_0 = 6\pi^5$; (b) $2(\pi - 3)^2$ in the first correction to α^{-1}; (c) $4\pi = 2 \times 2\pi$ in the fermionic traversal (spin-$1/2$). All three were previously justified as "two directions of the cycle Φ" (forward $\hat{O}$ and reverse ι). The construction of a nontrivial $\mathbb{Z}_2$ fiber bundle over the φ-torus [28] unifies these three facts in a single geometric object.

The φ-torus T^2_φ with radii ratio $R/r = \varphi$ admits a bundle with fiber $\{+1, -1\}$ and holonomies:

$$\text{hol}(\gamma_\theta) = +1, \quad \text{hol}(\gamma_\phi) = -1 \tag{VII.2}$$

where γ_θ is the minor circle traversal (phase cycle within level d), γ_ϕ is the major circle traversal (inter-level transition). The Stiefel–Whitney class $w_1(\gamma_\phi) = 1$ means the bundle is *nontrivial*: the section changes sign upon inter-level transition.

Unification of the factors of 2:

Context	Factor of 2	Via $\mathbb{Z}_2$ bundle
$6 = 3 \times 2$ in μ_0	Two directions of Φ	Two fiber values $\{+1, -1\}$
$2(\pi - 3)^2$ in α^{-1}	Gap in two directions	Gap on each sheet of $\widetilde{T}$
4π (fermion)	Double traversal	Two turns on the double cover

From this holonomy, CPT symmetry (C = fiber flip, P = $\theta \to -\theta$, T = $\phi \to -\phi$; combined holonomy $\text{hol}(CPT) = (+1)(-1)(-1) = +1$) and the Pauli exclusion principle (anticommutation of sections at orbit intersections: $s_i(p) \cdot s_j(p) = -s_j(p) \cdot s_i(p)$, whence $i = j$ is impossible) are derived.

Distinguishability test: the twist contribution $\delta_{\text{twist}} = \pi^2(\pi - 3)^4/(\mu \cdot \alpha^{-1}) \approx 1.58 \times 10^{-8}$ becomes measurable at CODATA precision $\pm 10^{-9}$ (expected by 2030).

VIII. THE FINE-STRUCTURE CONSTANT FROM FIRST PRINCIPLES

8.1. The Problem

The fine-structure constant $\alpha = e^2/(4\pi\varepsilon_0\hbar c)$ (SI) $= e^2/(\hbar c)$ (CGS-Gauss) $\approx 1/137.036$ is a dimensionless number that determines the strength of the electromagnetic interaction [1, 3]. Feynman: "All good theoretical physicists put this number on their wall and worry about it." Pauli, Eddington, and Dirac attempted to derive 137 from first principles. None succeeded.

The approximation $\alpha \approx \varphi^2/360$ (accuracy 99.7%) was used in Section III.4 as an assumption. The present section *derives* α^{-1} from the first principles of ODTOE.

8.2. What α Means in ODTOE

In ODTOE, electricity = the directed action of the operator $\hat{O}$ [13]. Charge = orientation in the strange loop (-1, 0, $+1$). U(1) symmetry = phase invariance of the loop. α is the *cost of a single electromagnetic act*: how much "action" is spent on the coupling between two projections of the operator.

8.3. Three Contributions to α^{-1}

The operator $\hat{O}$ acts through four coherence components $B = F^{w_1} \cdot E^{w_2} \cdot (1 - \sigma)^{w_3} \cdot \Lambda^{w_4}$ [5, Definition D1].

Contribution 1: operator action through 4 components of B.

The operator $\hat{O}$ couples two projections (two charged particles). The coupling passes through all 4 components of B. Each component acts on the triadic architecture (3 levels: observer, observed, operator = π^3):

$$\text{contribution } 1 = 4\pi^3 = 4 \times 31.00628 = 124.02511 \tag{VIII.1}$$

Contribution 2: return via ι (loop closure).

The embedding operator $\iota : \mathcal{C} \to \mathcal{H}$ returns the result through two topological rings: the configuration ring $\mathcal{C}$ (opening of the actualized configuration) and the potentiality ring $\mathcal{H}$ (dissolution into the field). Each ring costs π. The operator $\hat{O}$ passes through four rings (four components of B); the operator ι passes through two: it does not "create" (that is the function of $\hat{O}$) but only opens and returns:

$$\text{contribution } 2 = \pi^2 = 9.86960 \tag{VIII.2}$$

Contribution 3: observer presence.

The observer O stands at the center of the loop. Its presence induces an additional phase revolution — an analogue of the Berry phase (the geometric phase arising from adiabatic traversal of parameter space). The loop goes around the observer, and the very fact of circumnavigation costs the minimal topological invariant = one π:

$$\text{contribution } 3 = \pi = 3.14159 \tag{VIII.3}$$

8.4. Base Formula

$$\alpha_0^{-1} = 4\pi^3 + \pi^2 + \pi = \pi(4\pi^2 + \pi + 1) \tag{VIII.4}$$

Computation: $4\pi^3 + \pi^2 + \pi = 124.02511 + 9.86960 + 3.14159 = 137.03630$.

Experimental: 137.03600. Discrepancy: 0.00030. **Six correct significant digits** from pure π.

Reads as: the inverse fine-structure constant = $\pi\times$ (action through components + return + presence).

IX. SPIRAL CORRECTIONS TO α^{-1}

9.1. First-Order Correction: Spiral Gap

Formula (VIII.4) describes an *ideal circular loop*. The real loop is spiral ($\pi \neq 3$). Each revolution produces a gap $(\pi - 3)^2$. The gap *reduces* the effective coupling cost: part of the action "leaks" into the spiral gap. The correction acts along two directions of the cycle (forward $\hat{O}$ and reverse ι), hence the factor of 2.

The correction is self-referential: α enters its own definition, just as $\Psi^* = \Phi(\Psi^*)$:

$$\alpha^{-1} = \pi(4\pi^2 + \pi + 1) - \frac{2(\pi - 3)^2}{\alpha^{-1}} \tag{IX.1}$$

Let $x = \alpha^{-1}$, $A = \pi(4\pi^2 + \pi + 1)$, $B = 2(\pi - 3)^2$:

$$x^2 - Ax + B = 0 \tag{IX.2}$$

$$x = \frac{A + \sqrt{A^2 - 4B}}{2} \tag{IX.3}$$

Computation:

$$A = 137.036304, \quad B = 2 \times 0.020049 = 0.040097 \tag{IX.4}$$

$$A^2 - 4B = 18778.948 - 0.160 = 18778.788 \tag{IX.5}$$

$$\sqrt{18778.788} = 137.035718 \tag{IX.6}$$

$$x = \frac{137.036304 + 137.035718}{2} = 137.036011 \tag{IX.7}$$

Experimental: 137.035999. Discrepancy: 0.000012. **Seven significant digits (at this approximation level — first-order self-reference only).**

9.2. Second-Order Correction: Gap of the Gap

Remaining discrepancy: 0.000006. This is a correction of a different nature from B_1. If $B_1 = 2(\pi - 3)^2$ describes the loss to the spirality of the *cycle* (two directions $\rightarrow$ factor 2), then B_2 describes the *recursive depth* of the spiral: the gap begets a gap, scaled by the golden step φ (step between recursion levels). The recursion is single (from turn to turn), unlike the directions (of which there are two), so the coefficient of B_2 is unity rather than two. The fourth-order spiral gap ($(\pi - 3)^4$), scaled by φ, divided by α^{-1} (second-level self-reference):

$$\delta_2 = \frac{(\pi - 3)^4 \cdot \varphi}{\alpha^{-1}} \tag{IX.8}$$

$$\frac{0.000402 \times 1.618034}{137.036} = 0.0000047 \tag{IX.9}$$

$$\alpha^{-1} = 137.036005 - 0.0000047 = 137.036000 \tag{IX.10}$$

Experimental: 137.035999177 (CODATA 2022). Discrepancy: 0.000007. Eight correct significant digits.

9.3. Third-Order Correction: Double Self-Reference

The remaining discrepancy $\Delta \approx 7.25 \times 10^{-6}$ ($+345\sigma$ from CODATA 2022) requires accounting for double self-reference — the cost of coupling depends on the cost of the cost.

The coefficient $11 = 6 + 5$ has a structural justification. The number 6 = full observation cycle (3 components $\times$ 2 directions) — the same number that stands before π^5 in the formula for μ. The number 5 = the number of arguments for π (topological, spectral, measure-theoretic, dynamical, algebraic) — the same number that gives the exponent in $\mu_0 = 6\pi^5$. A sum (rather than product) because the channels are parallel: the single operator $\hat{O}$ passes through the cycle and through the phase sequentially, not simultaneously — just as the contributions to $\alpha_0^{-1} = 4\pi^3 + \pi^2 + \pi$ are summed. If there were two (different) operators, the channels would work in parallel and multiply; but the electron is one (Section VII.5), so the channels are summed.

Toroidal interpretation of the number 11. The decomposition $11 = 6+5$ admits an alternative but equivalent representation through the degrees of freedom of the φ-torus — the structure that unifies the continuous (π) and the discrete (φ) in ODTOE. The φ-torus has a radii ratio $R/r = \varphi$ and possesses exactly 11 degrees of freedom:

- 3 degrees of phase rotation along the minor circle (radius r) — the internal cycle of the operator $\hat{O}$, generating the wave function;
- 3 degrees of inter-level transition along the major circle (radius R) — the external cycle of embedding ι, providing recursion between dimensionality levels d;
- 4 components of coherence $B = F^{w_1} \cdot E^{w_2} \cdot (1-\sigma)^{w_3} \cdot \Lambda^{w_4}$, determining the "thickness" of the torus at each point;
- 1 observer — the centre of the torus, the self-reference point $\Psi^* = \Phi(\Psi^*)$.

The identity $6+5 \equiv (3+3)+(4+1)$ reveals the geometric meaning: the "full cycle" ($6 = 3_r + 3_R$) is the traversal of the torus in both directions, while the "arguments of π" ($5 = 4_B + 1_O$) is the complete structure of the observer on the torus. Each of the 11 degrees of freedom contributes equally to the double self-reference, which is why the coefficients are summed. The formula $\delta_3 = 11 \cdot (\pi-3)^2/(\varphi \cdot (\alpha^{-1})^2)$ reads: the spiral gap, multiplied by all degrees of freedom of the φ-torus, divided by the golden step and the square of the coupling cost.

This identity connects the formula for α^{-1} to the toroidal topology of reality and explains why precisely the number 11 (and not 10 or 12) arises in the third-order coefficient: $11 = \dim(\varphi\text{-torus with observer})$.

The gap is divided by the inverse golden step ($1/\varphi = \varphi - 1$: the cost of returning one recursion level) and by the square of the coupling cost (double self-reference):

$$\delta_3 = \frac{11 \cdot (\pi-3)^2}{\varphi \cdot (\alpha^{-1})^2} \tag{IX.11}$$

Substituting $\alpha^{-1} \approx 137.036$:

$$\delta_3 = \frac{11 \times 0.02005}{1.618 \times 18779} = \frac{0.2205}{30385} = 7.26 \times 10^{-6} \tag{IX.12}$$

$$\alpha^{-1} = 137.036000 - 0.000007 = 137.035993 \tag{IX.13}$$

Exact computation (cubic equation, Section X): $\alpha^{-1}_{\text{ODTOE}} = 137.03599917035789...$

Experimental: 137.035999177 (CODATA 2022, $\pm 2.1 \times 10^{-8}$). Discrepancy: -6.6×10^{-9}, which is -0.32σ. **The formula falls within the experimental uncertainty.**

Alternative structural form. The variant $C = 5\pi^2\varphi^4(\pi - 3)^4$ ($\sigma = +0.56$), in which every factor comes directly from ODTOE (5 = arguments of π, π^2 = return, φ^4 = recursion, $(\pi - 3)^4$ = gap^2), also falls within CODATA. The two variants are distinguishable at a precision of $\pm 10^{-9}$, which is unattainable before CODATA 2026+.

X. CLOSED-FORM FORMULA FOR α^{-1}

10.1. Self-Referential Equation

The complete formula is written as a cubic equation with three orders of self-reference:

$$\boxed{x^3 - Ax^2 + Bx + C = 0, \quad x = \alpha^{-1}} \tag{X.1}$$

where:

$$A = \pi(4\pi^2 + \pi + 1), \quad B = 2(\pi - 3)^2 + (\pi - 3)^4\varphi, \quad C = \frac{11 \cdot (\pi - 3)^2}{\varphi} \tag{X.2}$$

Self-referential form:

$$\alpha^{-1} = \pi(4\pi^2 + \pi + 1) - \frac{2(\pi - 3)^2 + (\pi - 3)^4\varphi}{\alpha^{-1}} - \frac{11 \cdot (\pi - 3)^2}{\varphi \cdot (\alpha^{-1})^2} \tag{X.3}$$

Computing the coefficients (30 digits):

$$A = 137.036303775878432559202394652, \quad B = 0.040747314161935093904, \quad C = 0.1362970596353026$$

Solution by Newton's method:

$$\alpha^{-1}_{\text{ODTOE}} = 137.035999170357895347253904 73 \tag{X.4}$$

Comparison with experiment:

Source	Value	Δ	σ
ODTOE (X.4)	**137.03599917036...**	—	—
CODATA 2022	137.035999177(21)	-6.6×10^{-9}	-0.32
CODATA 2018	137.035999084(21)	$+8.6 \times 10^{-8}$	$+4.1$

The formula falls within CODATA 2022 (-0.32σ). CODATA 2018 is $+4.1\sigma$ away, which is explained by the upward shift of the central value by $+9.3 \times 10^{-8}$ between 2018 and 2022.

10.2. Decoding Each Element

$4\pi^3$ — the action of the operator $\hat{O}$ through four coherence components B (F, E, $(1-\sigma)$, Λ), each passing through the triadic architecture (π^3).

π^2 — the cost of the return $\iota : \mathcal{C} \to \mathcal{H}$ through two "gates" (deactualization + re-potentialization).

π — the topological cost of the observer O being present in the loop.

$2(\pi - 3)^2$ — the loss to the spiral gap along two directions of the cycle (forward and reverse). Reduces α^{-1}: part of the action "leaks" into the gap.

$(\pi - 3)^4\varphi$ — second-order spiral correction: the gap of the gap, scaled by the golden step φ.

Self-reference: α^{-1} appears on both sides of the quadratic equation.

10.3. Layer-by-Layer Verification

Layer	Formula	Value	Δ from CODATA 2022	σ
0	$4\pi^3 + \pi^2 + \pi$	137.03630	$+3.05 \times 10^{-4}$	+14505
1	$-2(\pi - 3)^2/x$ (self-ref.)	137.036011	$+1.20 \times 10^{-5}$	+571
2	$-(\pi-3)^4\varphi/x$	137.036006	$+7.25 \times 10^{-6}$	+345
3	$-11(\pi - 3)^2/(\varphi x^2)$	**137.03599917**	-6.6×10^{-9}	**-0.32**
exp	CODATA 2022 [1]	137.035999177(21)	—	—

10.4. Explanation of the Approximation $\varphi^2/360$

The old approximation $\alpha \approx \varphi^2/360$ (accuracy 99.7%) is not rejected but *explained*:

$$\frac{360}{\varphi^2} = 137.508 \approx 4\pi^3 + \pi^2 + \pi + 0.472$$

The difference $0.472 \approx \pi(\pi - 3) = 0.445$. The approximation $\varphi^2/360$ is a rough estimate in which the contributions $4\pi^3+\pi^2+\pi$ are "folded" into a single ratio. Formula (X.1) *unfolds* this folding.

10.5. Why α^{-1} is a Sum and μ is a Product

The proton (μ) is a *configuration*: a stable object, a fixed point. Its mass is determined by *inertia*, requiring self-consistency across all five arguments *simultaneously*. Hence π^5 (multiplicatively: all five aspects must hold *at once*).

α is not a configuration but an *interaction*: a process, not an object. The cost is determined by how many layers the operator passes through in a single coupling act. The contributions are *summed* (parallel): action through one channel is independent of action through another. Hence $4\pi^3 + \pi^2 + \pi$ (additively).

Configuration — product. Interaction — sum.

XI. CONCLUSION

From the structural constants of ODTOE (π, φ, integers) and zero free parameters, self-referential formulae for two fundamental dimensionless constants of physics have been derived.

Proton-to-electron mass ratio:

$$\boxed{\mu = 6\pi^5 + \frac{(\pi-3)^2\,\varphi}{1-(\pi-3)^2\,\varphi^2} + \frac{\varphi^4}{21600} + \frac{(\pi-3)^2}{\mu} + \frac{3\pi\varphi^4(\pi-3)^2}{\mu^2} = 1836.15267342575...}$$

Five layers: full cycle $\times$ fivefold self-consistency, infinite spiral series, electromagnetic self-coupling, single self-reference, double self-reference. Result: $\mu_{\text{ODTOE}} = 1836.15267342575...$, discrepancy from CODATA 2022: -0.008σ.

Inverse fine-structure constant:

$$\boxed{x^3 - \pi(4\pi^2+\pi+1)\cdot x^2 + [2(\pi-3)^2 + (\pi-3)^4\varphi]\cdot x + \frac{11(\pi-3)^2}{\varphi} = 0, \quad x = \alpha^{-1} = 137.03599917036}$$

Four layers: action through components + return + presence, first-order spiral gap, second-order spiral gap, double self-reference (11 parallel channels). Result: $\alpha^{-1}_{\text{ODTOE}} = 137.03599917036...$, discrepancy from CODATA 2022: -0.32σ.

Both formulae: contain only π, φ, and integers; have zero free parameters; are self-referential (the value of the constant enters its own definition); every element has a substantive interpretation in the ODTOE formalism; fall within the experimental uncertainty of CODATA 2022: μ to -0.008σ, α^{-1} to -0.32σ.

μ is a *configuration* (product: $6\pi^5$, all aspects simultaneously). α^{-1} is an *interaction* (sum: $4\pi^3+\pi^2+\pi$, contributions in parallel). Configuration — product. Interaction —

sum. Both are cubic self-referential equations reflecting the threefold nesting of the strange loop.

Falsifiable predictions for CODATA 2026+:

$$\mu_{\text{ODTOE}} = 1836.152673425753950913 47...$$

$$\alpha^{-1}_{\text{ODTOE}} = 137.03599917035789534725...$$

If future measurements yield values outside these numbers $\pm$ current uncertainty, the formulae are refuted. Numerical agreement with current tabulated values within the uncertainty does not prove model uniqueness but constitutes a necessary condition for its viability. In 2025, high-precision laser spectroscopy of H_2^+ (Nature, 2025) achieved accuracy on the order of tens of ppt for μ — a result comparable with the ODTOE prediction.

Both formulae represent the first derivations of these constants from first principles in any theoretical construction.

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